Last name _____

First name _____

LARSON—MATH 756—HOMEWORK WORKSHEET 11
Knuth's *Sandwich Theorem* paper

Polytopes.

1. Let $\hat{x}_1, \ldots, \hat{x}_k$ be points in $\mathbb{R}^n$. Define the convex hull $\mathcal{P}$ of $\{\hat{x}_i\}$:

   - (Implicitly) Let $\hat{x}_i \in \mathcal{P}$ and if $\hat{x}, \hat{y} \in \mathcal{P}$ then $t\hat{x} + (1-t)\hat{y} \in \mathcal{P}$ (for $t \in [0,1]$).
   - (Explicitly) Let $\mathcal{P}$ be the set of convex combinations of $\{\hat{x}_i\}$, that is:

   $$\mathcal{P} = \{t_1\hat{x}_1 + \ldots + t_k\hat{x}_k | t_i \geq 0 \text{ and } \sum_{i=1}^{k} t_i = 1\}.$$

   Show these definitions of $\mathcal{P}$ are equivalent.

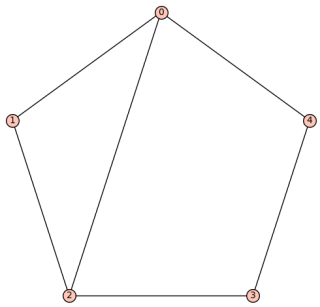
The ϑ_1 invariant.

2. For each of the following graphs G, estimate ϑ_1 by finding upper and lower bounds. Use our theorem $\vartheta \leq \vartheta_1$ to find lower bounds and orthogonal representations $\{veca_v\}$ of G to find upper bounds.

   (a) The path graph P_3.

   (b) The paw graph.

   (c) The cycle C_5 with an additional chord.



The ϑ_2 invariant.

3. Let G be the paw graph. Show $\vartheta_2(G) = 2$ by proving a lower bound for ϑ_2 and by finding a feasible matrix for G with largest eigenvalue 2.

