Last name _____

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LARSON—MATH 756—HOMEWORK WORKSHEET 10
Knuth's *Sandwich Theorem* paper

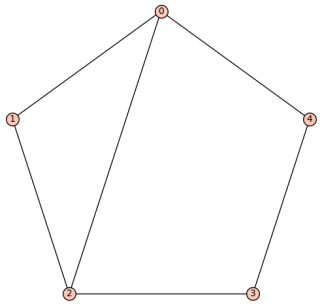
Definitions from Knuth

- STAB(G) is defined to be the *convex hull* of the stable labels in G.
- $\alpha = \max\{\hat{x} \cdot \hat{1} | \hat{x} \in STAB(G)\}$.
- TH(G)=$\{\hat{x} \geq 0 | \sum_{v \in V} x_v c(\hat{a}_v) \leq 1$ for every orthonormal labeling $\{\hat{a}_v\}$ of $G\}$
- $\vartheta = \max\{\hat{x} | \hat{x} \in TH(G)$.
- QSTAB(G) is defined to be $\{\vec{x} \geq 0 | \sum_{v \in Q} x_v \leq 1,$ for every clique Q of $G\} \subseteq \mathcal{R}^n$.
- $\kappa = \max\{\hat{x} \cdot \hat{1} | \hat{x} \in QSTAB(G)\}$.
- $\bar{\omega}$ is the *clique covering number*, the minimum number of cliques needed to partition the vertex set of a graph.
- We **proved**: for any graph
$$\alpha \leq \vartheta \leq \kappa \leq \bar{\omega}.$$

Problems

Use the definitions, the theorem above, together with the "monotonicity" results in Knuth's paper to calculate or estimate, α, ϑ, κ, and $\bar{\omega}$ for the following graphs. **Explain**.

1. The cycle graph C_5.
2. The cycle graph C_5 with one chord.



3. The cycle graph C_5 with two chords.

