Last name _____

First name _____

LARSON—MATH 756—HOMEWORK WORKSHEET 09
Knuth's *Sandwich Theorem* paper

Definitions from Knuth

- STAB(G) is defined to be the *convex hull* of the stable labels in G.
- $\alpha = \max\{\hat{x} \cdot \hat{1} | \hat{x} \in STAB(G)\}$.
- TH(G)=$\{\hat{x} \geq 0 | \sum_{v \in V} x_v c(\hat{a}_v) \leq 1$ for every orthonormal labeling $\{\hat{a}_v\}$ of $G\}$
- $\vartheta = \max\{\hat{x} | \hat{x} \in TH(G)$.
- QSTAB(G) is defined to be $\{\vec{x} \geq 0 | \sum_{v \in Q} x_v \leq 1,$ for every clique Q of $G\} \subseteq \mathcal{R}^n$.
- $\kappa = \max\{\hat{x} \cdot \hat{1} | \hat{x} \in QSTAB(G)\}$.

General Directions

For each graph:

1. Find at least 3 stable labelings for G, including all labeling corresponding to maximum stable sets.
2. Find at least 3 orthonormal representations $\{\hat{a}_v\}$ for G.
3. Find the *cost* $c(\hat{a}_v)$ of each vector $\hat{a}_v$ in each orthonormal representation.
4. Find the stable labelings $\hat{x}$ that satisfy the defining condition $\sum_{v \in V} x_v c(\hat{a}_v) \leq 1$ of TH(G) for your othonormal representations.
5. Find the maximum value $\hat{x} \cdot \hat{1}$ for the feasible $\hat{x}$ vectors from the last problem.
6. Can you find any vectors $\hat{x}$ in TH(G) that give a larger value of $\hat{x} \cdot \hat{1}$ than what you've already found?
7. Use your computations to estimate ϑ.
8. Find a sample of vectors in QSTAB(G).
9. Estimate κ.

Problems

1. Do these steps for the path P_3.
2. Do these steps for the cycle graph C_5.
3. Do these steps for the bow-tie graph (two triangles fused at a center vertex).