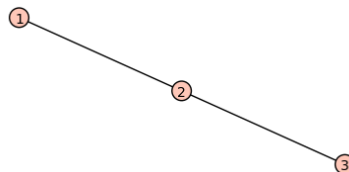


Last name \_\_\_\_\_

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LARSON—MATH 756—HOMEWORK WORKSHEET 08  
Knuth's *Sandwich Theorem* paper



**Real Spectral Theorem Review**

1. Find an *adjacency matrix*  $A$  for  $P_3$ .
2. Find all the eigenvalues, and corresponding normalized eigenvectors for  $P_3$ .
3. Find an orthogonal matrix  $Q$  and diagonal matrix  $D$  so that  $A = QDQ^T$ . Check all the details.

**Characteristic labelings, STAB and QSTAB**

For Knuth a *stable set* is a synonym for *independent set*. For a graph  $G$  and vertex set  $V = \{v_1, \dots, v_n\}$  ( $n = |V|$ ), a *characteristic labeling* of a set  $S \subseteq V$  is a vector  $\hat{x}$  in  $\mathcal{R}^n$  with  $x_v = 1$  if  $v \in S$  and  $x_v = 0$  otherwise.

4. List all of the stable sets in  $P_3$ , and find their corresponding characteristic labelings (these are *stable labelings*).

$\text{STAB}(G)$  is defined to be the *convex hull* of the stable labels in  $G$ . So, if  $\hat{x}$  and  $\hat{y}$  are in  $\text{STAB}(G)$  then  $t\hat{x} + (1-t)\hat{y}$  is in  $\text{STAB}(G)$  for every  $t \in [0, 1]$  (intuitively this means: for every pair of vectors in  $\text{STAB}(G)$  the points on the line between them are also in  $\text{STAB}(G)$ ).

5. Find examples of non-integer coordinate vectors in  $\text{STAB}(P_3)$ .
6. List all of the cliques in  $P_3$ , and find their corresponding characteristic labelings (these are *clique labelings*).

$\text{QSTAB}(G)$  is defined to be  $\{\vec{x} \geq 0 \mid \sum_{v \in Q} x_v \leq 1, \text{ for every clique } Q \text{ of } G\} \subseteq \mathcal{R}^n$ .

7. Check that all the **stable** labelings for  $P_3$  are in  $\text{QSTAB}(P_3)$ .
8. Find examples of non-integer coordinate vectors in  $\text{QSTAB}(P_3)$ .
9. Show: for any graph  $G$ ,  $\text{STAB}(G) \subseteq \text{QSTAB}(G)$ .
10. Show: for any graph  $G$ ,  $\text{QSTAB}(G)$  is a convex set in  $\mathcal{R}^n$ .