Last name ______________________

First name ______________________

LARSON—MATH 756—HOMEWORK WORKSHEET 03
Bounds for the Independence Number of a Graph

1. Show: $\alpha + \omega \leq n + 1$.

For a connected graph G, a subset D of the vertex set is *dominating* (or a *dominating set*) if every vertex not in D is adjacent to *some* (at least one) vertex in D; if the graph is not connected a set D of vertices is dominating if it dominates each connected component of the graph: in other words, each vertex is either in D or is adjacent to some vertex in D. The *domination number* γ of a graph is the cardinality of a minimum dominating set.

2. Show: $\alpha \geq \gamma$.

The *degree* $d(v)$ of a vertex v is the number of vertices v is adjacent to. The maximum degree Δ is the largest degree; in other words, $\Delta = \max\{d(v) : v \in V\}$. The minimum degree δ is the smallest degree of the graph.

3. Show: $\alpha \leq n - \delta$.
4. Show : $\alpha \geq \frac{n}{\Delta+1}$.
5. Show: $\alpha \leq n - \frac{m}{\Delta}$, where m is the *size* (number of edges).

Bonus

For a connected graph, the *distance* $d(v, w)$ from vertex v to vertex w is the length (number of edges) in a shortest path from v to w. The *eccentricity* $e(v)$ of a vertex v is the maximum distance to any other vertex; formally, this is $e(v) = \max\{d(v, w) : w \in V\}$. The *radius* r of a (connected) graph is the minimum eccentricity of any of its vertices.

6. Show: the number of vertices of a maximum induced path in a graph is at least $2r-1$. Then show the corollary: $\alpha \geq r$.