

Last name _____

First name _____

LARSON—MATH 756—HOMEWORK WORKSHEET 02
Independence Number Theorems

Problems.

For each problem,

- define relevant terms,
- provide a careful, convincing, complete proof, and
- provide an example.

1. Let G_1 and G_2 be the *components* of a graph G . Prove: $\alpha(G) = \alpha(G_1) + \alpha(G_2)$.
2. If v is a pendant vertex in a graph G then there is a maximum independent set of G containing v .
3. (Gallai Identity) Prove, for any graph, that $\alpha + \beta = n$, where n is the order of the graph, and β is the covering number.
4. Prove: for any graph G , that $\alpha(G) = \omega(\bar{G})$, where ω is the clique number of the graph.
5. Prove: the chromatic number χ of a graph G equals the *clique covering number* $\bar{\omega}$ of its complement $\bar{G}$.
6. Prove: $n - 2\mu \leq \alpha$, for any graph, where μ is the *matching number*.
7. Prove: $\alpha \leq n - \mu$ for any graph.