Last name _____

First name _____

LARSON—MATH 756—HOMEWORK WORKSHEET 01
Tarjan-Trojanowski Algorithm

Main Idea: a vertex v of a graph G is either in a maximum independent set or it is not.

Note: If a vertex v is in a maximum independent set I then none of its *neighbors* $N(v)$ can be in I.

Notation:

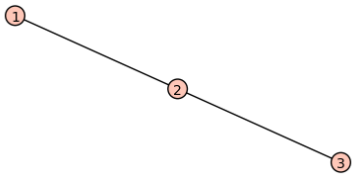
- We use $N[v]$ to denote the *closed neighborhood* consisting of v and its neighbors (so $N[v] = N(v) \cup \{v\}$).
- For a graph G and subset of vertices S, we use $G - S$ to denote the subgraph induced on the remaining vertices when the vertices in S are removed. So, $G - S = G[V(G) \backslash S]$.

These facts yield a recursive condition, for any vertex v:

$$\alpha(G) = \max\{\alpha(G - v), \alpha(G - N[v]) + 1\}.$$

Problems:

1. Prove this recursive condition.
2. Apply the Tarjan-Trojanowski idea to find a maximum independent set in P_3 (the path on 3 vertices), and $\alpha(P_3)$.



3. Let G be the Petersen graph. Apply the Tarjan-Trojanowski idea to find a maximum independent set in G (the path on 3 vertices), and $\alpha(G)$.