Last name _____

First name _____

LARSON—MATH 756—CLASSROOM WORKSHEET 18
Knuth—Sandwich Theorem

Review

1. Why is STAB(G) $\subseteq$ QSTAB(G)?
2. What is the *cost* $c(\hat{a}_v)$ of a (unit) vector $\hat{a}_v$ (for $v \in V$) in an orthonormal labeling of a graph G?
3. Show: for a graph G, any independent set S, and any orthonormal labeling $\{a_v\}$, $\sum_{v \in S} c(\hat{a}_v) \leq 1$.
4. What is TH(G) for a graph G?
5. Why is TH(G) "obviously" convex?

New

1. Find TH(G) for the paw graph.

2. Show: STAB(G) $\subseteq TH(G)$, for any graph G.

3. What is Knuth's first definition of ϑ?

4. Why is $\alpha(G) = \max\{\hat{x} \cdot \hat{1} | \hat{x} \in STAB(G)\}$?

5. What is Knuth's proof that $\alpha \leq \vartheta$?

6. Why is $TH(G) \subseteq QSTAB(G)$, for any graph G?

7. What is κ? (Knuth defines $\kappa(G) = \max\{\hat{x} \cdot \hat{1} | \hat{x} \in QSTAB(G)\}$).

Monotonicity

8. What is Knuth's definition of $G \subseteq G'$?

9. Why does $G \subseteq G'$ imply $STAB(G') \subseteq STAB(G)$?

10. Why does $G \subseteq G'$ imply $TH(G') \subseteq TH(G)$?

11. Why does $G \subseteq G'$ imply $QSTAB(G') \subseteq QSTAB(G)$?