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First name _____

LARSON—MATH 756—CLASSROOM WORKSHEET 06
Berge and Hall's Theorems

Review

1. We proved that, for any graph, $\omega \leq \chi$. What does this fact imply about α ?
2. Can you think of any class of graphs where $\alpha = \bar{\omega}$? (Can you characterize the graphs where equality holds?)
3. Can you think of any class of graphs where $\alpha = n - 2\mu$? (Can you characterize the graphs where equality holds?)
4. Can you think of any class of graphs where $\alpha = n - \mu$? (Can you characterize the graphs where equality holds?)
5. If M is a matching in a graph G , what is an M -alternating path in G ?
6. If M is a matching in a graph G , what is an M -augmenting path in G ?

New

1. If M is a matching in a graph G and $P = v_1, v_2, \dots, v_{2k}$ is an M -augmenting path in G find a matching M' such that $|M'| > |M|$.
2. (Berge's Theorem) Prove: if G is a graph with matching M , M is maximum if and only if G has no M -augmenting path.

3. What doesn't Berge's Theorem lead to a non-exponential algorithm for finding a maximum matching in a general graph?
4. How can Berge's Theorem be used to find a non-exponential algorithm for finding a maximum matching in a *bipartite* graph?
5. What is Hall's Theorem?
6. What is Hall's Condition?
7. Why is Hall's Theorem true?