Last name _____

First name _____

LARSON—MATH 756—CLASSROOM WORKSHEET 05
Matchings and Berge's Theorem

Review

1. Describe a polynomial-time algorithm for finding a maximum independent set of vertices in a tree.
2. What is a *proper coloring* of a graph? (and what is the chromatic number χ?)
3. What can you say about the *color classes* of a proper coloring?
4. Why does the chromatic number $\chi(G)$ of a graph G equal the minimum number of cliques of $\bar{G}$ that include all vertices (the clique covering number $\bar{\omega}$)?
5. Prove: for any graph $\omega \leq \chi$.
6. Prove: for any graph $\alpha \leq \bar{\omega}$.
7. What is a *matching* in a graph?
8. What is an algorithm for finding a maximum matching in a graph?
9. What is the *matching number* μ?
10. Prove: for any graph, $n - 2\mu \leq \alpha$.
11. Prove: for any graph, $\alpha \leq n - \mu$.

New

1. We proved that, for any graph, $\omega \leq \chi$. What does this fact imply about α?

2. Can you think of any class of graphs where $\alpha = \bar{\omega}$? (Can you characterize the graphs where equality holds?)

3. Can you think of any class of graphs where $\alpha = n - 2\mu$? (Can you characterize the graphs where equality holds?)

4. Can you think of any class of graphs where $\alpha = n - \mu$? (Can you characterize the graphs where equality holds?)

5. If M is a matching in a graph G, what is an M-*alternating* path in G?

6. If M is a matching in a graph G, what is an M-*augmenting* path in G?

7. If M is a matching in a graph G and $P = v_1, v_2, \ldots, v_{2k}$ is an M-augmenting path in G find a matching M' such that $|M'| > |M|$.

8. (Berge's Theorem) Prove: if G is a graph with matching M, M is maximum if and only if G has no M-augmenting path.