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First name _____

LARSON—MATH 756—CLASSROOM WORKSHEET 04
Relatives of the Independence Number

Review

1. (Gallai Identity) Prove, for any graph, that $\alpha + \beta = n$, where n is the order of the graph.
2. What is a *clique* in a graph (and the clique number ω)?
3. What is the complement $\bar{G}$ of a graph?
4. Prove: for any graph G , that $\alpha(G) = \omega(\bar{G})$.

New

1. Describe a polynomial-time algorithm for finding a maximum independent set of vertices in a tree.
2. What is a *proper coloring* of a graph? (and what is the chromatic number χ ?)
3. What can you say about the *color classes* of a proper coloring?
4. Why does the chromatic number $\chi(G)$ of a graph G equal the minimum number of cliques of $\bar{G}$ that include all vertices (the clique covering number $\bar{\omega}$)?

5. Prove: for any graph $\omega \leq \chi$. (What does this fact imply about α ?)

6. Prove: for any graph $\alpha \leq \bar{\omega}$.

7. What is a *matching* in a graph?

8. What is an algorithm for finding a maximum matching in a graph?

9. What is the *matching number* μ ?

10. Prove: for any graph, $n - 2\mu \leq \alpha$.

11. Prove: for any graph, $\alpha \leq n - \mu$.