

Last name _____

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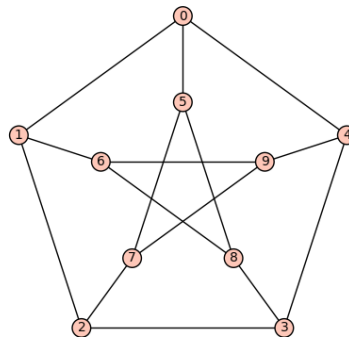
LARSON—MATH 756—CLASSROOM WORKSHEET 03
Relatives of the Independence Number

Review

1. Prove: if v is a pendant vertex in a graph G then there is a maximum independent set of G containing v .
2. Find a formula for α for a star with n vertices.
3. Find a formula for α for a path with n vertices.
4. What is a *vertex cover* of a graph (and the vertex covering number β)?

New

1. (Gallai Identity) Prove, for any graph, that $\alpha + \beta = n$, where n is the order of the graph.
2. Find the vertex covering number β of the Petersen graph.



3. What is a *clique* in a graph (and the clique number ω)?

4. Find the clique number ω for the Petersen graph.
5. What is the complement $\bar{G}$ of a graph?
6. Prove: for any graph G , that $\alpha(G) = \omega(\bar{G})$.
7. Describe a polynomial-time algorithm for finding a maximum independent set of vertices in a tree.
8. What is a *proper coloring* of a graph? (and what is the chromatic number χ ?)
9. What can you say about the *color classes* of a proper coloring? Why does the chromatic number $\chi(G)$ of a graph H equal the minimum number of cliques of $\bar{G}$ that include all vertices (the clique covering number *omega*)?