LARSON—MATH 756—Fall 2026
Outline & Annotated Bibliography

Getting Started.

1. **Basics**. Independence number, algorithms, formulas, graph classes.
2. **Hall-related Theorems**. Berge, Hall, Kőnig.
3. **Integer Programming**. Optimization formulation of independence-number, linear programming and cutting planes.
4. **Lovász' paper**. Orthogonal representations, Lovász original definition and the key inequality.
5. **Lovász-theta equivalents**. From Knuth's Sandwich Theorem paper.

Bibliography.

1. **Graph Theory Books**. The first graph theory book was published in German in 1939. An english translation [21] published in 1990. The first french graph theory book was Berge's in 1958 [4]. A still-influential english text is Bondy and Murty's from 1976 [6]. For an interesting history of graph theory see [5].
2. **NP-Completeness of MIS**. MIS (or equivalently Maximum Clique or Minimum Cover) was one of the original 21 problems that Karp proved is NP-complete in 1972 [19].
3. **Tarjan-Trojanowski Algorithm**. Tarjan and Trojanowski's 1977 algorithm is in [38]. A very good practical practical algorithm is `Cliquer` due to Ostergard [32].
4. **Lovasz' Theta**. The original Lovasz paper is [28]. Knuth wrote an interesting 1993 paper with several equivalent definitions [20].
5. **Matching, König's Theorem, Hungarian Method**. Berge's 1957 theorem is in [4]. The Hungarian Method was first presented by Kuhn in [22]. The history of König's Theorem, as well as proofs of several interesting equivalent theorems is in Lovasz and Plummer's classic book on matching theory [29].
6. **König-Egervary Graphs**. The original papers were due to Deming and Sterboul, both in 1979 [12] [37].
7. **Integer Programming and Fractional Independence**. Balinki's Lemma is usually cited as [3]. The Nemhauser-Trotter Theorem is in [31]. The Picard-Queyranne Theorem is in [34]. The Independence Decomposition Theorem (IDT) is in [24]. Connections between integer programming approaches and IDT are is [25].
8. **Critical Independent Sets**. These were first introduced by C.Q. Zhang in [39]. Ageev made the first connection between critical independent sets and linear programming in [1]. Algorithms were reported in [23] and [11].
9. **Eigenvalue Bounds**. Cvetkovic proved his 1971 eigenvalues upper bound for the independence number in [9]. The main tool is the super-useful Cauchy Interlacing Theorem (where and when is this from?). A very useful compilation of this and other graph eigenvalue results is [10].

10. **Perfect Graphs**. The Perfect Graph Conjecture is due to Berge at the same time [36]. The Weak Perfect Graph Theorem was proved by Lovasz in 1972 [30]. The Strong Perfect Graph Theorem was proved by Chudnovsky, Robertson, Seymour and Thomas [8]. A polynomial-time recognition algorithm was proved shortly after [**?**].

11. **Cutting Planes & Alpha-Critical Graphs**. The literature on stable set (or vertex packing) polytopes is vast. Start with this [35] comprehensive and useful reference filled with historical information.

12. **Alpha-critical Graphs**. Critical graphs were first studied by Dirac [13]. Lovasz reports that α-critical graphs were studied by Erdős and Gallai in the early 1960s (he cites [15]). The theorem that incident edges are contained in an induced odd cycle is due to Andrasfai [2]. Lovasz writes about α-critical graphs in many places. His first substantial results are in [27]

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