Last name _____

First name _____

LARSON—OPER 731—HOMEWORK h06
Test 1 Review

Concepts For each concept, give a definition and an example.

1. What is a *linear program*?
2. What is an *integer program*?
3. What is a *mixed integer program*?
4. What is an *assignment problem*?
5. What is a *feasible solution* to an LP?
6. What is a *optimal solution* to an LP?
7. What is a *certificate of optimality* for an LP?
8. What is the *canonical form* for an LP?
9. What is a an s-t-path?
10. What is an st-cut in a graph?
11. What is a *matching* in a graph?
12. What is a *polyhedron*?
13. What is an *extreme point* in a polyhedron?

Theorems

14. What is the *Fundamental Theorem of Linear Programming*?
15. What is the *Weak Duality Theorem*?

Problems Explain everything. As scientists it is never enough to write answers. They must be communicated—convincingly—to others.

Consider the IP:

maximize: $z = x_1 + x_2 + x_3$

$$\begin{array}{rrcrcrcl} & x_1 & + & x_2 & & & \leq & 1 \\ \text{subject to:} & x_1 & & & + & x_3 & \leq & 1 \\ & & & x_2 & + & x_3 & \leq & 1 \end{array}$$

$x_1, x_2, x_3 \in \{0, 1\}$.

16. Find a feasible solution.
17. Suppose we replace the integer requirement with $x_1, x_2, x_3 \geq \mathbb{0}$ (so now we have an LP). Explain why the LP optimum must be at least as large as the IP optimum.

18. Write this LP in the form $\max\{c^T x : Ax \leq b, x \geq \mathbb{0}\}$. What are the A, b, c?

19. Find the *dual* of this LP.

20. Find a feasible solution for the dual and explain what it tells you about the optimum of the primal LP.

21. What is the *rank* of A?

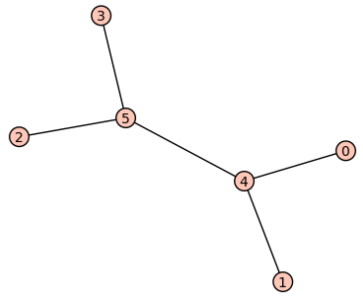
22. Write this LP in *standard equality form*.

23. Find a *basis* for this LP.

24. Find a *basic feasible solution*.

25. Suppose a model includes a constraint $x_4 \geq |x_1 + x_2|$. If we want an LP model, how can we rewrite this constraint for our LP?

26. If the LP $\max\{c^T x : Ax \leq b, x \geq \mathbb{0}\}$ is infeasible, what could you do to *prove* that it is infeasible?

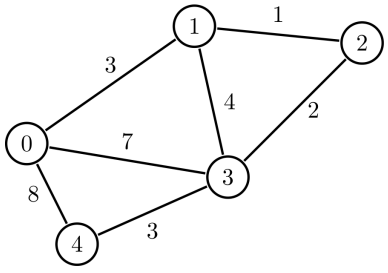


27. Write an IP to model finding a maximum matching in this graph. Explain.

28. Write an IP to model finding a minimum vertex cover in this graph. Explain.

29. What is the "obvious" algorithm to find a minimum vertex cover in a graph. Explain why it would take an exponential number of steps (as a function of the number of vertices)?

30. Let P be an st-path in a graph. Why must every edge of P be contained in some st-cut?



31. What is the (unique) minimum weight 1-3 path in this graph?

32. Write an IP to find a minimum weight 1-3 path in this graph. Explain.

33. Find a feasible solution.

34. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find A^{-T}.