LARSON—OPER 731—CLASSROOM WORKSHEET 24
**Max Flow Min Cut!**

Concepts

- (Sec. 3.1) *dual LP, Weak duality theorem.*

- (Sec. 4.3) *complementary slackness*, cone, *cone of tight constraints.*

- (Sec. 4.4) *Farkas's Lemma.*

- (Sec. 5.1) *primal-dual algorithm.*

- (Sec. 5.3) *directed graph, flow, flow balance, flow value, capacity, max-flow min-cut.*
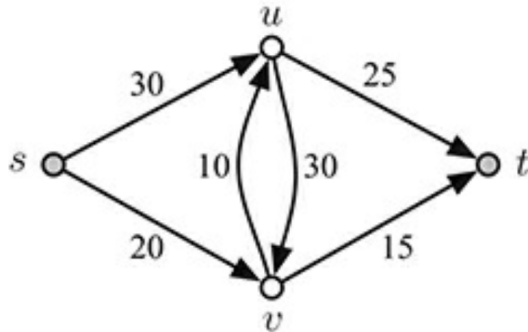
1. What is a *directed graph*?

2. What is the vertex-arc incidence matrix of a directed graph?

3. What is a *totally unimodular matrix*?

4. Why is the vertex-edge incidence matrix of a directed graph totally unimodular?

5. What is an *s-t flow*? What is the *value* of a flow?

$$f_x(q) := \sum (x_a \ : \ a \in \delta^+(q)) - \sum (x_a \ : \ a \in \delta^-(q)) = 0,$$

6. What does the notation in the *flow balance* equation mean?
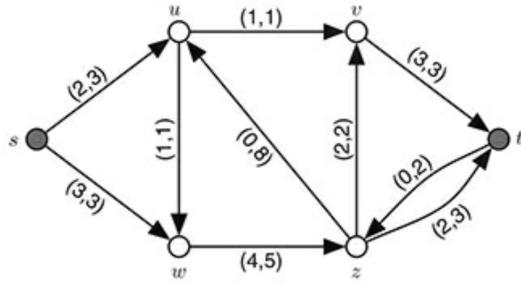
7. The first numbers on each edge are flow values and the second numbers are edge capacities. Do the flow values indicate a valid flow? What is the value of this flow?

8. Can you find a flow with a larger value in this network? If not, can you prove that this flow is maximum?

9. Model the maximum s-t flow problem for this network.

10. What is an s-t cut? What is the *capacity* of an s-t cut?

11. Can you find a minimum cut in this network?

12. (**Claim:**). If a max-flow problem has integer capacities and an optimal soultion, then there is an optimal *integer flow*.