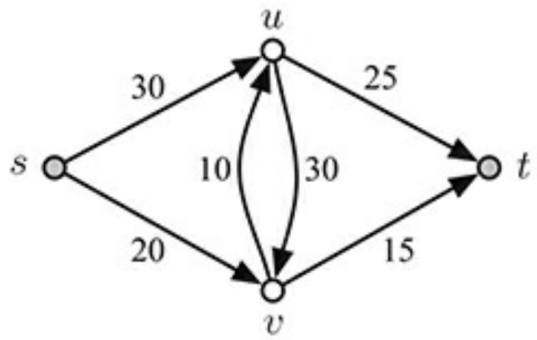
Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 23
Max Flow Min Cut!

Concepts

- (Sec. 3.1) *dual LP*, *Weak duality theorem*.
- (Sec. 4.3) *complementary slackness*, cone, *cone of tight constraints*.
- (Sec. 4.4) *Farkas's Lemma*.
- (Sec. 5.1) *primal-dual algorithm*.
- (Sec. 5.3) *directed graph*, *flow*, *flow balance*, *flow value*, *capacity*, *max-flow min-cut*.



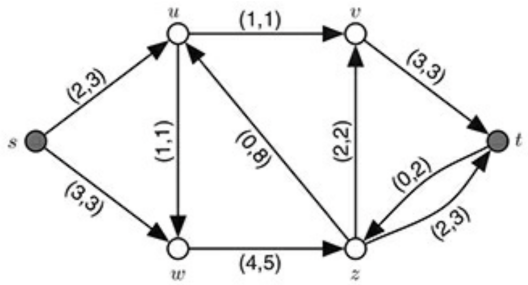
1. What is a *directed graph*?
2. What is the vertex-arc incidence matrix of a directed graph?
3. What is a *totally unimodular matrix*?

4. Why is the vertex-edge incidence matrix of a directed graph totally unimodular?

5. What is an *s-t flow*? What is the *value* of a flow?

$$f_x(q) := \sum(x_a : a \in \delta^+(q)) - \sum(x_a : a \in \delta^-(q)) = 0,$$

6. What does the notation in the *flow balance* equation mean?



7. The first numbers on each edge are flow values and the second numbers are edge capacities. Do the flow values indicate a valid flow? What is the value of this flow?

8. Can you find a flow with a larger value in this network? If not, can you prove that this flow is maximum?

9. Model the maximum s-t flow problem for this network.

10. What is an *s-t cut*? What is the *capacity* of an s-t cut?

11. Can you find a minimum cut in this network?