Last name ______________________

First name ______________________

LARSON—OPER 731—CLASSROOM WORKSHEET 20
The Primal-Dual Algorithm

Concepts

- (Sec. 3.1) *dual LP*, *Weak duality theorem*.
- (Sec. 4.3) *complementary slackness*, cone, *cone of tight constraints*.
- (Sec. 5.1) *primal-dual algorithm*.

Geometry of Optimal Solutions

1. (**Claim**:) Let $\bar{x}$ be a feasible solution to $max\{c^T x : Ax \leq b\}$. Then $\bar{x}$ is optimal if and only if and only if c is in the cone of tight constraints for $\bar{x}$.

Primal-Dual Algorithm

2. We will revisit our minimum cost perfect matching example (C_4 with edge costs: $2, 3, 4, 5$). We used "reduced costs" to find a dual-feasible maximum of 6 (so the minimum cost is no more than 6). How can we use complementary slackness to *prove* that 6 is primal-optimal?

3. What is the *set cover* problem?

4. What is the *edge cover* problem? How is it an instance of the set cover problem?

5. What is the relationship between the vertex cover problem and the vertex packing problem?

6. The authors claim that the vertex cover problem is an instance of the set cover problem. Is it?

Algorithm 5.6 Primal–dual algorithm for set-cover

Input: Elements $\mathscr{U}$, sets $\mathscr{S}$, costs c_S for all $S \in \mathscr{S}$.

Output: A collection $\mathscr{C} \subseteq \mathscr{S}$ such that $\mathscr{U} \subseteq \cup_{S \in \mathscr{C}} S$, and a feasible dual y for (5.2).

1: $y = \mathbb{0}$, $\mathscr{C} = \emptyset$

2: **while** $\exists\, e \in \mathscr{U}$ that is not covered by any set in $\mathscr{C}$ **do**

3: Increase y_e as much as possible, maintaining feasibility of y for (5.2)

4: Let S be a tight set cover e

5: Add S to $\mathscr{C}$

6: **end while**

7: **return** $\mathscr{C}$ and feasible dual y

7. Try this algorithm on our set cover example.