Last name ______________________

First name ______________________

LARSON—OPER 731—CLASSROOM WORKSHEET 16
Complementary Slackness!

Concepts

- (Sec. 2.4) *basis*, *basic* variable, *nonbasic* variable, *basic solution*, *basic feasible solution*, *canonical form*.
- (Sec. 2.8) *hyperplane*, *halfspace*, *line*, *line segment*, *convex*, *polyhedron*, *tight* inequality, *extreme point*.
- (Sec. 3.1) *dual LP*, *Weak duality theorem*.
- (Sec. 4.3) *complementary slackness*, cone, *cone of tight constraints*.

1. What is *complementary slackness*? What is the *Complementary Slackness Theorem*?

Geometry of Optimal Solutions

2. What is the *cone* of vectors $a^{(1)}, a^{(2)}, \ldots, a^{(k)}$ in $\mathbb{R}^n$.

3. Find the cone of vectors $a^{(1)} = (2, -1)^T$, $a^{(2)} = (3, 1)^T$, $a^{(3)} = (2, 1)^T$ in $\mathbb{R}^2$.

4. Check that $\bar{x} = (2, 1)^T$ is feasible for the primal linear program:

   max $(\frac{3}{2}, \frac{1}{2})x$

   subject to:

   $$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} x \le \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$$

   $x \ge \mathbb{O}$

5. Identify which constraints are tight for $\bar{x}$. Let $J(\bar{x})$ be the corresponding row indices.

6. What is the *cone of tight constraints* for $\bar{x}$ for polyhedron $(P) = \{x : Ax \leq b\}$ in this example?

7. Find the dual LP.

8. Suppose $\bar{x}$ were optimal for the primal LP. What would the complementary slackness conditions require for an optimal $\bar{y}$ for the dual LP?

9. Find a dual feasible $\bar{y}$ that we can use to prove that $\bar{x}$ is optimal for the primal LP.

10. (**Claim**:) Let $\bar{x}$ be a feasible solution to $max\{c^T x : Ax \leq b\}$. Then $\bar{x}$ is optimal if and only if and only if c is in the cone of tight constraints for $\bar{x}$.