Last name ______________________

First name ______________________

LARSON—OPER 731—CLASSROOM WORKSHEET 16
Complementary Slackness!

Concepts

- (Sec. 2.4) *basis*, *basic* variable, *nonbasic* variable, *basic solution*, *basic feasible solution*, *canonical form*.
- (Sec. 2.8) *hyperplane*, *halfspace*, *line*, *line segment*, *convex*, *polyhedron*, *tight* inequality, *extreme point*.
- (Sec. 3.1) *dual LP*, *Weak duality theorem*.
- (Sec. 4.3) *complementary slackness*, cone, *cone of tight constraints*.

1. What is *complementary slackness*? What is the *Complementary Slackness Theorem*?

2. Find the dual for following (primal) optimization problem:

   max $(5,3,5)x$

   subject to:

   $$\begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 1 \end{pmatrix} x \leq \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$$

   $x \geq \mathbb{O}$

3. What are the complementary slackness conditions for an optimal solution to this primal-dual pair?

4. Check that $\bar{x} = (1,-1,1)^T$ is primal-feasible and $\bar{y} = (0,2,1)^T$ is dual-feasible.

5. Check that $\bar{x}$ and $\bar{y}$ are optimal by verifying the complementary slackness conditions.

Geometry of Optimal Solutions

6. What is the *cone* of vectors $a^{(1)}, a^{(2)}, \ldots, a^{(k)}$ in $\mathbb{R}^n$.

7. Find the cone of vectors $a^{(1)} = (2, -1)^T$, $a^{(2)} = (3, 1)^T$, $a^{(3)} = (2, 1)^T$ in $\mathbb{R}^2$.

8. Check that $\bar{x} = (2, 1)^T$ is feasible for the linear program:

   max $(\frac{3}{2}, \frac{1}{2})x$

   subject to:

   $$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} x \leq \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$$

   $x \geq \mathbb{O}$

9. Identify which constraints are tight for $\bar{x}$. Let $J(\bar{x})$ be the corresponding row indices.

10. What is the *cone of tight constraints* for $\bar{x}$ for polyhedron $(P) = \{x : Ax \leq b\}$ in this example?