

Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 11
Duality!

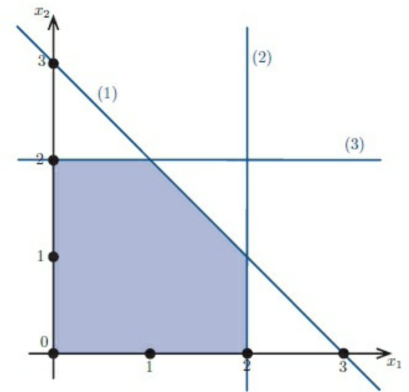
Concepts

- (Sec. 2.4) *basis, basic variable, nonbasic variable, basic solution, basic feasible solution, canonical form.*
- (Sec. 2.8) *hyperplane, halfspace, line, line segment, convex, polyhedron, tight inequality, extreme point.*
- (Sec. 3.1) *dual LP, Weak duality theorem.*

Review

1. **Claim:** Let A be a matrix with linearly independent rows and b be a vector. Let $P = \{x : Ax = b, x \geq \mathbb{0}\}$ and let $\bar{x} \in P$. Then \bar{x} is an extreme point of P if and only if \bar{x} is a basic feasible solution of $Ax = b$.

$$\begin{array}{ll} \max & (c_1, c_2)x \\ \text{s.t.} & \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} x \leq \begin{pmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}. \end{array} \quad \begin{array}{l} (1) \\ (2) \\ (3) \\ (4) \\ (5) \end{array}$$



2. Find all extreme points.
3. Find all the basic feasible solutions for $Ax = b$ in the example above.

Duality

4. Consider the LP: $\max\{c^T x : Ax \leq b, x \geq \mathbb{0}\}$.

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ -1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 20 \\ 18 \\ 8 \end{pmatrix} \quad c = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

Find the dual. Find feasible solutions for the primal and dual. Use these to estimate the optimal value of the primal objective function.

5. What is the *Weak Duality Theorem*?

6. We will consider a shortest-path LP and investigate how the dual can be interpreted.