Last name ______________________

First name ______________________

LARSON—OPER 731—CLASSROOM WORKSHEET 10
Duality!

Concepts

- (Sec. 2.4) *basis*, *basic* variable, *nonbasic* variable, *basic solution*, *basic feasible solution*, *canonical form*.
- (Sec. 2.8) *hyperplane*, *halfspace*, *line*, *line segment*, *convex*, *polyhedron*, *tight* inequality, *extreme point*.
- (Sec. 3.1) *dual LP*.

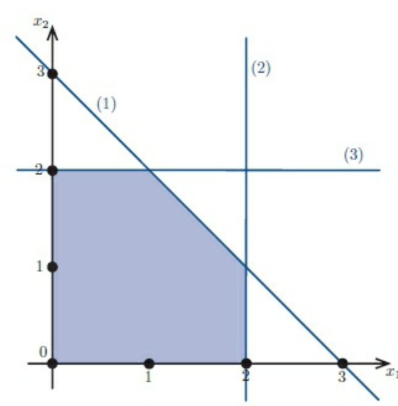
Review

1. What is the *line* through points $x^{(1)}$ and $x^{(2)}$ in $\mathbb{R}^n$?
2. What is the *line segment* through points $x^{(1)}$ and $x^{(2)}$ in $\mathbb{R}^n$?
3. When is a set $C \subseteq \mathbb{R}^n$ *convex*?
4. **Claim**: Halfspaces are convex.
5. **Claim**: The intersection of halfspaces is convex.
6. **Claim**: Polyhedra are convex.
7. What is an *extreme point* of a polyhedron?
8. When is an inequality $\alpha^T x = \beta$ *tight* for a point $\bar{x}$.
9. **Notation**: What is $A^{=}x \leq b^{=}$ for a point $\bar{x}$?

Geometry

$$\begin{array}{ll} \max & (c_1,\ c_2)x \\ \text{s.t.} & \end{array}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} x \leq \begin{pmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}. \qquad \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \\ (5) \end{array}$$



10. **Claim**: For a polyhedron $P = \{x \in \mathbb{R}^n : Ax \leq b\}$, $x \in \mathbb{R}^n$, and $A^{=}x \leq b^{=}$ tight for $\bar{x}$, $\bar{x}$ is an extreme point of P if and only if $rank(A^{=}) = n$.

11. **Claim**: Let A be a matrix with linearly independent rows and b be a vector. Let $P = \{x : Ax = b, x \geq \mathbb{O}\}$ and let $\bar{x} \in P$. Then $\bar{x}$ is an extreme point of P if and only if $\bar{x}$ is a basic feasible solution of $Ax = b$.

Duality

12. Consider the LP: $\max\{c^T x : Ax \leq b, x \geq \mathbb{O}\}$.

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ -1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 20 \\ 18 \\ 8 \end{pmatrix} \quad c = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

Find the dual. Find feasible solutions for the primal and dual. Use these to estimate the optimal value of the primal objective function.

13. We will consider a shortest-path LP and investigate how the dual can be interpreted.