Last name ______________________

First name ______________________

LARSON—OPER 731—CLASSROOM WORKSHEET 09
The Geometry of Linear Programs

Concepts

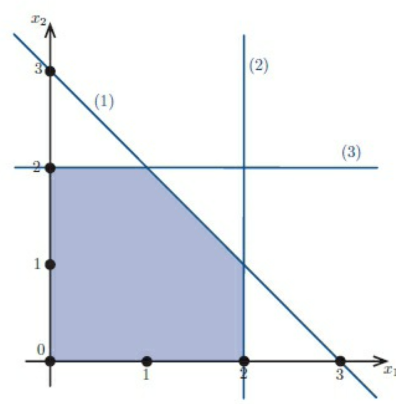
- (Sec. 2.4) *basis*, *basic* variable, *nonbasic* variable, *basic solution*, *basic feasible solution*, *canonical form.*
- (Sec. 2.8) *hyperplane*, *halfspace*, *line*, *line segment*, *convex*, *polyhedron*, *tight* inequality, *extreme point*

Review

1. What is a *hyperplane* in $\mathbb{R}^n$?
2. What is a *halfspace* in $\mathbb{R}^n$?
3. Why are hyperplanes in $\mathbb{R}^n$ $(n-1)$-dimensional?
4. What is a *polyhedron* in $\mathbb{R}^n$?

Geometry

$$\begin{aligned}&\max \quad (c_1,\ c_2)x\\ &\text{s.t.}\\ &\begin{pmatrix}1 & 1\\ 1 & 0\\ 0 & 1\\ -1 & 0\\ 0 & -1\end{pmatrix} x \le \begin{pmatrix}3\\ 2\\ 2\\ 0\\ 0\end{pmatrix}. \qquad \begin{matrix}(1)\\ (2)\\ (3)\\ (4)\\ (5)\end{matrix}\end{aligned}$$



5. What is the *line* through points $x^{(1)}$ and $x^{(2)}$ in $\mathbb{R}^n$?

6. What is the *line segment* through points $x^{(1)}$ and $x^{(2)}$ in $\mathbb{R}^n$?

7. When is a set $C \subseteq \mathbb{R}^n$ *convex*?

8. **Claim**: Halfspaces are convex.

9. **Claim**: The intersection of halfspaces is convex.

10. **Claim**: Polyhedra are convex.

11. What is an *extreme point* of a polyhedron?

12. When is an inequality $\alpha^T x = \beta$ *tight* for a point $\bar{x}$.

13. **Notation**: What is $A^= x \leq b^=$ for a point $\bar{x}$?

14. **Claim**: For a polyhedron $P = \{x \in \mathbb{R}^n : Ax \leq b\}$, $x \in \mathbb{R}^n$, and $A^= x \leq b^=$ tight for $\bar{x}$, $\bar{x}$ is an extreme point of P if and only if $rank(A^=) = n$.

15. **Claim**: Let A be a matrix with linearly independent rows and b be a vector. Let $P = \{x : Ax = b, x \geq \mathbb{O}\}$ and let $\bar{x} \in P$. Then $\bar{x}$ is an extreme point of P if and only if $\bar{x}$ is a basic feasible solution of $Ax = b$.