Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 08
The Geometry of Linear Programs

Concepts

- (Sec. 2.4) *basis*, *basic* variable, *nonbasic* variable, *basic solution*, *basic feasible solution*, *canonical form*.

- (Sec. 2.8) *hyperplane*, *halfspace*, *line*, *line segment*, *convex*, *polyhedron*, *tight* inequality, *extreme point*

1. If we have a feasible LP, in SEF, how can we find an initial basis, and initial basic feasible solution?

Geometry

$$\begin{array}{ll}
\max & (c_1, c_2)x \\
\text{s.t.} &
\end{array}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} x \leq \begin{pmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 0 \end{pmatrix}.
\begin{array}{l} (1) \\ (2) \\ (3) \\ (4) \\ (5) \end{array}$$
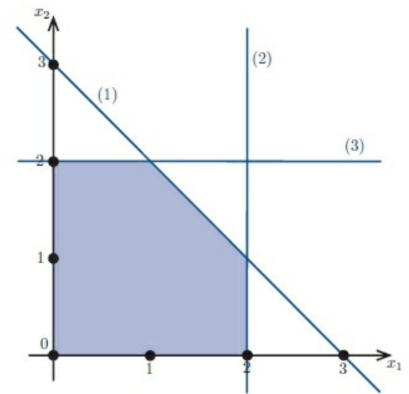
2. What is a *hyperplane* in $\mathbb{R}^n$?

3. What is a *halfspace* in $\mathbb{R}^n$?

4. Why are hyperplanes in $\mathbb{R}^n$ $(n-1)$-dimensional?

5. What is a *polyhedron* in $\mathbb{R}^n$?

6. What is the *line* through points $x^{(1)}$ and $x^{(2)}$ in $\mathbb{R}^n$?

7. What is the *line segment* through points $x^{(1)}$ and $x^{(2)}$ in $\mathbb{R}^n$?

8. When is a set $C \subseteq \mathbb{R}^n$ *convex*?

9. **Claim**: Halfspaces are convex.

10. **Claim**: The intersection of halfspaces is convex.

11. **Claim**: Polyhedra are convex.

12. What is an *extreme point* of a polyhedron?

13. When is an inequality $\alpha^T x = \beta$ *tight* for a point $\bar{x}$.

14. **Notation**: What is $A^= x \le b^=$ for a point $\bar{x}$?