Last name _____

First name _____

LARSON—OPER 731—CLASSROOM WORKSHEET 02
Modeling

Concepts

- (Sec. 1.1) *formulation*, *feasible* solution.
- (Sec 1.2) *linear function*, *linear constraint*, *linear program.*
- (Sec. 1.3) *integer program*, *mixed integer program.*

1. What is the history of linear programming (linear optimization)? Who uses it? Where is it taught? How is it useful for theoretical discrete mathematics?

2.

1.3.1 Assignment problem

Our friends at WaterTech are once again looking to us for help. The company faces the following problem: there is a set of J jobs that need to be handled urgently. The company has selected I of its most trusted employees to handle these jobs. Naturally, the skill sets of these employees differ, and not all of the jobs are equally well handled by all of the employees. From past experience, management knows the number of hours c_{ij} each worker $i \in I$ is expected to take in order to complete any of the jobs $j \in J$. The following table gives an example for a case with $|J| = 4$ jobs and $|I| = 4$ employees:

	Jobs			
Employees	1	2	3	4
1	3	5	1	7
2	8	2	2	4
3	2	1	6	8
4	8	3	3	2

For instance, the table says that $c_{3,4} = 8$, i.e. employee 3 would take eight hours to finish job 4. WaterTech wants to assign jobs to employees with the conditions that:

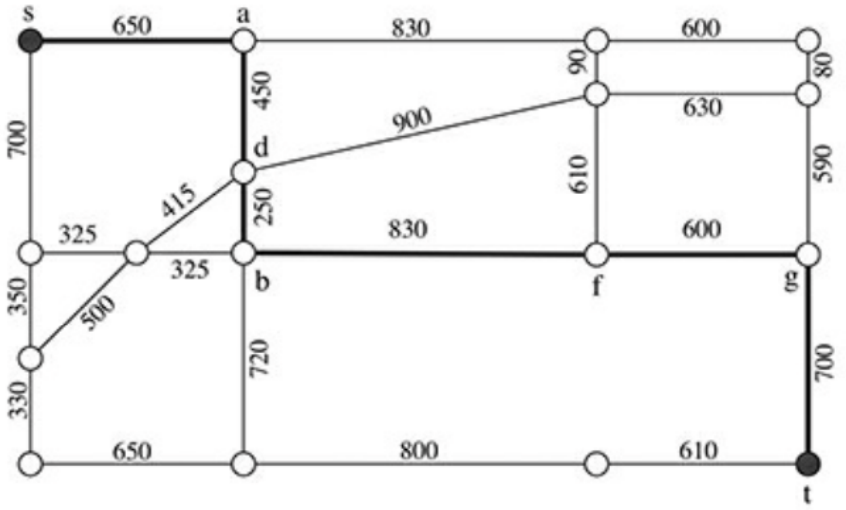
(1) each employee $i \in I$ is assigned exactly one job $j \in J$,
(2) each job $j \in J$ is assigned to exactly one employee $i \in I$.

Both of these conditions can only be satisfied when $|I| = |J|$. Naturally, we want to find such an assignment that minimizes the total expected amount of time needed to process all jobs J. A feasible solution would be to assign job k

(a) Formulate this problem: define decision variables, write any constraints, and write an objective function.

(b) Can you find a feasible solution?

(c) Can you find an optimal solution? (If so, how can you *prove* it?).



3. What is a an s-t-path?

4. What is the *length* $c(P)$ of an st-path P with edge-lenths $(c_e : e \in P)$?

5. What is a minimum st-path?

6. How is this a discrete optimization problem?

7. For a graph $G = (V, E)$, and vertices $U \subseteq V$, what is $\delta(U)$?

8. What is an *st-cut*?