Last name ______________________

First name ______________________

LARSON—MATH 610—HOMEWORK h11
Gram-Schmidt Example

Given any basis $v_1, \ldots, v_n$ of $\mathbb{R}^n$, we can *construct* an orthonormal basis $u_1, \ldots, u_n$ of $\mathbb{R}^n$ (this method applies to any finite-dimensional vector space, but our example will be in $\mathbb{R}^3$).

The method is the following:

1. Let $u_1 = \frac{1}{||v_1||} v_1$.
2. After $j-1$ iterations, let $u_j' = v_j - \langle v_j, u_1 \rangle u_1 - \ldots - \langle v_j, u_{j-1} \rangle u_{j-1}$.
3. Let $u_j = \frac{1}{||u_j'||} u_j'$.
4. Repeat.

Problem

Let $v_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

1. Check that v_1, v_2, v_3 is a basis for $\mathbb{R}^3$.
2. Apply the Gram-Schmidt method to v_1, v_2, v_3 to produce an orthonormal basis u_1, u_2, u_3 for $\mathbb{R}^3$. Use the dot product as your inner product for this method. Show all your steps.
3. Check that u_1, u_2, u_3 is an orthonormal basis for $\mathbb{R}^3$. What are all the things you need to check?