Last name ________________________

First name ________________________

LARSON—MATH 610–HOMEWORK WORKSHEET 07
Test 1 Review.

General Instructions

1. **You should know the following definitions, and corresponding examples, for the test. Write out careful definitions and problem solutions. Turn these in at test time**.
2. Write up a **neat** assignment on a **new sheet** of paper. (Do not cram your answers between the lines).
3. **Number** your problems so that it is easy to see what work matches the assigned problems.
4. Remember to **give examples** (you do not understand a concept unless you can provide an example of it).

Definitions. Write each definition **and give an example**.

1. What is the *conjugate transpose* (or *adjoint*) A^* of a matrix $A \in \mathbb{M}_{m\times n}$?
2. What is a *symmetric* matrix?
3. What is a *Hermitian* matrix?
4. What is a *vector space*?
5. What is a *subspace* of a vector space?
6. What is an *orthogonal* matrix?
7. What is a *linearly independent* list of vectors?
8. What is the *rank* of a list of vectors?
9. What is the *dimension* of a vector space?
10. What is a *linear transformation*? What is $\mathcal{L}(\mathcal{V}, \mathcal{W})$?
11. What is the *outer product* of vectors $\hat{x}$ and $\hat{y}$?
12. What is the *nullity* of a matrix A?
13. What is an *inner product* on an $mathbbR$ or $\mathbb{C}$-vector space?

    Theorems. State each theorem carefully, and **give an example**.

14. **Pivot Column Decomposition**.
15. **Full Rank Factorization**.

16. **Dimension Theorem** (for linear transformations).

17. **Rank-Nullity** Theorem (for matrices).

18. **Pythagorean Theorem** in an inner product space.

19. **Cauchy-Schwartz** inequality.

20. **Gram-Schmidt**.

Problems. Carefully **explain** your answer.

21. Find an example of a Hermitian matrix that is not symmetric.

22. Argue: if a list of vectors is linearly dependent then at least one of the vectors must be a linear combination of the others.

23. Argue: no linearly independent list of vectors can contain the $\hat{0}$ vector.

24. Find a basis for the column space of:

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

25. Find the pivot column decomposition for:

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

26. How does Full Rank Factorization show that, if $A \in \mathbb{M}_{n\times n}$ is invertible, then the columns of A are linearly independent?

27. What is a formula for AB in terms of the rows of A. Give an example.

28. Write a formula for the product AB in terms of an *outer product* of the columns of A and the rows of B. Give an example.

29. Find the *projection* $\hat{x}$ of $\hat{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ on $\hat{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Check that $\hat{v} - \hat{x}$ is *orthogonal* to $\hat{u}$.

30. Show: If $\mathcal{V}$ be a finite-dimensional inner product space, and $\beta = \hat{u}_1, \ldots, \hat{u}_n$ be an orthonormal basis for $\mathcal{V}$ and let $\hat{v} \in \mathcal{V}$, then $\hat{v} = \sum_{i=1}^{n} \langle \hat{v}, \hat{u}_i \rangle \hat{u}_i$.

31. $\hat{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \hat{v}_2 = \hat{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \hat{v}_3 = \hat{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is a linearly independent list in the inner product space $\mathbb{R}^3$ with the dot product. Find an orthonormal list $\hat{u}_1, \hat{u}_2, \hat{u}_3$ with $Span(\hat{v}_1) = Span(\hat{u}_1)$, $Span(\hat{v}_1, \hat{v}_2) = Span(\hat{u}_1, \hat{u}_2)$, and $Span(\hat{v}_1, \hat{v}_2, \hat{v}_3) = Span(\hat{u}_1, \hat{u}_2, \hat{u}_3)$.

32. Argue: Every finite-dimensional vector space has an orthonormal basis.