Last name ______________________

First name ______________________

LARSON—MATH 610—HOMEWORK h06
Test 1 Review

Concepts For each concept, give a definition and an example.

1. What is $\mathbb{R}^\infty$?
2. What is a *subspace* of a vector space V?
3. What is $U_1 \oplus U_2$ for subspaces U_1, U_2 of a vector space V?
4. What is a *linear combination* of vectors $v_1, \ldots, v_m$ (over a field $\mathbb{F}$)?
5. What is the *span* of vectors $v_1, \ldots, v_m$ (over a field $\mathbb{F}$)?
6. What is a *linearly independent* list of vectors?
7. What is a *linearly dependent* list of vectors?
8. What is a *basis* of a vector space?
9. What is the *dimension* of a finite-dimensional vector space?
10. What is a *linear map*?
11. What is $\mathcal{L}(V, W)$?
12. Let $T \in \mathcal{L}(V, W)$. What is the *null space* of T?
13. What is a vector space *isomorphism*?
14. What is an *eigenvalue* of $T \in \mathcal{L}(V)$?
15. What is an *eigenvector* of $T \in \mathcal{L}(V)$?

Theorems. State (if needed), and provide a proof.

16. (**Basis Criterion**). A list $v_1 \ldots ; v_n$ of vectors in a vector space V is a basis of V if and only if every $v \in V$ can be written uniquely in the form $v = a_1 v_1 + \ldots + a_n v_n$ ($a_i \in \mathbb{F}$).

17. (**Linearly independent list extends to a basis**). Every linearly independent list of vectors in a finite-dimensional vector space can be extended to a basis of the vector space.

18. (**Linear map lemma**.) If $v_1, \ldots, v_n$ is a basis for vector space V and $w_1, \ldots, w_n$ is a basis for vector space W then there is a unique linear map $T : V \to W$ with $Tv_i = w_i$.

19. **Rank-Nullity Theorem**.

20. **Linearly independent eigenvectors**: Suppose $T \in \mathcal{L}(V)$. Every list of eigenvectors of T corresponding to distinct eigenvalues is linearly independent.

Problems Explain everything. As scientists it is never enough to write answers. They must be communicated—convincingly—to others.

21. **Show:** $\mathbb{R}^\infty$ is a vector space.

22. **Show:** The span of vectors $v_1, \ldots, v_m$ in V is a subspace of V?

23. Let $T \in \mathcal{L}(V, W)$. **Show:** *null* T is a subspace of V.

24. Let $T \in \mathcal{L}(V, W)$. **Show:** T is injective if and only if *null* $T = \{0\}$.

25. For $T \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$ with $T(x, y) = (x + 3y, 2x + 5y, 7x + 9y)$, find $\mathcal{M}(T)$.

26. **Show**: For vector spaces V, W, U, and linear maps $S : U \to V$ and $T : V \to W$, define TS and show that it is linear.

27. **Show**: λ is an eigenvalue of T if and only if $T - \lambda I$ is not injective.

28. Suppose $T \in \mathcal{L}(\mathbb{R}^2)$, with $T(z, w) = (w, z)$. Find all eigenvalues and eigenvectors of T.

29. Suppose $P \in \mathcal{L}(V)$, with $P^2 = P$ and λ is an eigenvalue of P. Show: λ is 0 or 1.