Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 40
Inertia, Interlacing, Cvetkovic.

(Chp. 5). *inner product*, *inner product space*, $\langle\ \rangle$, *orthogonal* vectors, $\perp$, $||\cdot||$.
(Chp. 6). *orthogonal basis.*
(Chp. 7). *unitary matrix*, *unitarily similar.*
(Chp. 8). $U^{\perp}$, $V = U \oplus U^{\perp}$, *orthonormal projection*, $P_{\mathcal{U}}\hat{v}$.
(Chp. 9). eigenvalue, eigenvector, eigenpair, nullity, annihilating polynomial, spectrum, Gershgorin disk, *diagonally dominant.* **(Chp. 15)**. positive semi-definite, square root $A^{\frac{1}{2}}$.
(Chp. 18). Rayleigh quotient.

Chp. 18 of Garcia & Horn, Matrix Mathematics

Review:

1. (**Cauchy's Interlacing Theorem**) We proved: if $A \in \mathbb{M}_n(\mathbb{C})$ and A is Hermitian with eigenvalues $\lambda_1 \leq \ldots \leq \lambda_n$, and B is a principal $(n-1)\times(n-1)$ submatrix of A with eignenvalues $\mu_1 \leq \ldots \leq \mu_{n-1}$ then

$$\lambda_1 \leq \mu_1 \leq \cdots \leq \lambda_{n-1} \leq \mu_{n-1} \leq \lambda_n.$$

2. The full version is for a principal $m \times m$ submatrix B with eigenvalues $\mu_1 \leq \ldots \leq \mu_m$. Then:

$$\lambda_k \leq \mu_k \leq \lambda_{k+(n-m)}.$$

1. (**An application**) What is the *independence number* of a graph?

(**Cvetkovic's Theorem**) If G is a graph with independence number, and adjacency matrix A, α then

$$\alpha \leq \text{ \# of non-negative eigenvalues of } A.$$

Also,

$$\alpha \leq \text{ \# of non-positive eigenvalues of } A.$$

2. How does the Interlacing Theorem imply Cvetkovic's Theorem?

3. Argue: If $A \in \mathbb{M}_n(\mathbb{R})$ is a symmetric matrix and $B \in \mathbb{M}_n(\mathbb{R})$, then BAB^T is symmetric (so both A and BAB^T have real eigenvalues).

(**Inertia Theorem**) If $A \in \mathbb{M}_n(\mathbb{R})$ is a symmetric matrix and $B \in \mathbb{M}_n(\mathbb{R})$ is invertible, then A and BAB^T have the same number of positive eigenvalues (and they have the same number of negative eigenvalues).

4. What is an example?