Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 28
Eigenvalues.

(Chp. 5). *inner product*, *inner product space*, $\langle\ \rangle$, *orthogonal* vectors, $\perp$, $||\cdot||$.
(Chp. 6). *orthogonal basis.*
(Chp. 7). *unitary matrix*, *unitarily similar.*
(Chp. 8). $U^{\perp}$, $V = U \oplus U^{\perp}$, *orthonormal projection*, $P_{\mathcal{U}}\hat{v}$.
(Chp. 9). eigenvalue, eigenvector, eigenpair, nullity, annihilating polynomial, spectrum.

Review:

1. Let $A \in \mathbb{M}_n(\mathbb{F})$. A can be viewed as a linear transformation for $\mathbb{F}^n$ to $\mathbb{F}^n$. Let $\hat{v}_1, \ldots, \hat{v}_n$ be a basis for $\mathbb{F}^n$. The following are equivalent:
   (a) A is invertible.
   (b) A is a bijection.
   (c) $A\hat{v}_1, \ldots, A\hat{v}_n$ is a basis for $\mathbb{F}^n$.

Chp. 9 of Garcia & Horn, Matrix Mathematics

(**Theorem 9.2.15. Existence of eigenvalue**). Every square complex matrix has an eigenvalue.

1. Why is this theorem true?

(**Theorem 9.3.5.**). Let $(\lambda_1, \hat{x}_1), \ldots, (\lambda_d, \hat{x}_d)$ be eigenpairs of $A \in \mathbb{M}_n$, in which $\lambda_1, \ldots, \lambda_d$ are all distinct. Then $\hat{x}_1, \ldots, \hat{x}_d$ are linearly independent.

2. What is an example?

3. Why is this theorem true?

4. What is the *spectrum* of a square matrix?

5. What can we say about the spectrum of $A = \begin{bmatrix} 1 & .1 & .2 \\ .3 & 2 & .3 \\ .4 & .5 & 7 \end{bmatrix}$?

6. For $A = [a_{ij}] \in \mathbb{M}_n$ $(n \geq 2)$ what is the *Gershgorin disk* $G_k(A)$?

7. What is **Gershgorin's Disc Theorem**?