Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 27
Eigenvalues.

(Chp. 5). *inner product*, *inner product space*, $\langle\ \rangle$, *orthogonal* vectors, $\perp$, $||\cdot||$.
(Chp. 6). *orthogonal basis*.
(Chp. 7). *unitary matrix*, *unitarily similar*.
(Chp. 8). $U^\perp$, $V = U \oplus U^\perp$, *orthonormal projection*, $P_{\mathcal{U}}\hat{v}$.
(Chp. 9). eigenvalue, eigenvector, eigenpair, nullity, annihilating polynomial, spectrum.

Chp. 9 of Garcia & Horn, Matrix Mathematics

(**Theorem 9.1.16**). Let $A \in \mathbb{M}_n$ and let $\lambda \in \mathbb{C}$. The following statements are equivalent.

(a) λ is an eigenvalue of A.

(b) $A\hat{x} = \lambda\hat{x}$ for some non-zero $\hat{x} \in \mathbb{C}^n$.

(c) $(A - \lambda I)\hat{x} - \hat{0}$ has a non-trivial solution (and thus, $nullity(A - \lambda I) > 0$).

(d) $rank(A - \lambda I) < n$.

(e) $A - \lambda I$ is not invertible.

(f) $A^T - \lambda I$ is not invertible.

(g) λ is an eigenvalue of A^T.

1. Why does (d) imply (e)?

2. Why does (e) imply (a)?

(**Theorem 9.2.15. Existence of eigenvalue**). Every square complex matrix has an eigenvalue.

3. Why is this theorem true?

(**Theorem 9.3.5.**). Let $(\lambda_1, \hat{x}_1), \ldots, (\lambda_d, \hat{x}_d)$ be eigenpairs of $A \in \mathbb{M}_n$, in which $\lambda_1, \ldots, \lambda_d$ are all distinct. Then $\hat{x}_1, \ldots, \hat{x}_d$ are linearly independent.

4. What is an example?

5. Why is this theorem true?

6. What is the *spectrum* of a square matrix?

7. For $A = [a_{ij}] \in \mathbb{M}_n$ $(n \geq 2)$ what is the *Gershgorin disk* $G_k(A)$?