

Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 26
Adjoint Operators.

Concepts & Notation

- (Chp. 6) *dot product, inner product, inner product space, norm, orthogonal representation, Cauchy-Schwartz, orthonormal list, Gram-Schmidt, orthogonal complement, orthogonal projection.*
- (Chp. 7) *adjoint, conjugate transpose.*

1. What is a *linear functional*?
2. What is the *Riesz Representation Theorem*?
3. (**Claim**) The adjoint of a linear map on an inner product space is linear.

7.7 Null space and range of T^*

Suppose $T \in \mathcal{L}(V, W)$. Then

(a) $\text{null } T^* = (\text{range } T)^\perp;$

(b) $\text{range } T^* = (\text{null } T)^\perp;$

(c) $\text{null } T = (\text{range } T^*)^\perp;$

(d) $\text{range } T = (\text{null } T^*)^\perp.$

- 4.
5. What is the *conjugate transpose* A^* of an $m \times n$ matrix?

6. **(Claim)** If V, W are finite-dimensional inner-product spaces with orthonormal bases e_1, \dots, e_n and f_1, \dots, f_m and $T \in \mathcal{L}(V, W)$, then the matrix of T^* equals the conjugate transpose of the matrix of T .

7. What is a *self-adjoint* linear operator (on an inner product space)?

8. **(Claim)** Eigenvalues of self-adjoint operators are real.

9. What is the *real spectral theorem*?