Last name ______________________

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LARSON—MATH 610—CLASSROOM WORKSHEET 25
Adjoint Operators.

Concepts & Notation

- (Chp. 6) *dot product*, *inner product*, *inner product space*, *norm*, *orthogonal representation*, *Cauchy-Schwartz*, *orthonormal list*, *Gram-Schmidt*, *orthogonal complement*, *orthogonal projection*.
- (Chp. 7) *adjoint*, *conjugate transpose*.

1. What is a *linear functional*?

2. What is the *Riesz Representation Theorem*?

3. Let V, W be finite-dimensional inner product spaces and $T \in \mathcal{L}(V, W)$. What is the *adjoint* T^* of T? Why does the adjoint exist?

4. (**Claim**) The adjoint of a linear map on an inner product space is linear.

5.

7.7 Null space and range of T^*

Suppose $T \in \mathcal{L}(V, W)$. Then

(a) $\operatorname{null} T^* = (\operatorname{range} T)^\perp$;

(b) $\operatorname{range} T^* = (\operatorname{null} T)^\perp$;

(c) $\operatorname{null} T = (\operatorname{range} T^*)^\perp$;

(d) $\operatorname{range} T = (\operatorname{null} T^*)^\perp$.

6. What is the *conjugate transpose* A^* of an $m \times n$ matrix?

7. (**Claim**) If V, W are finite-dimensional inner-product spaces with orthonormal bases $e_1, \ldots, e_n$ and $f_1, \ldots, f_m$ and $T \in \mathcal{L}(V, W)$, then the matrix of T^* equals the conjugate transpose of the matrix of T.

8. What is a *self-adjoint* linear operator (on an inner product space)?

9. (**Claim**) Eigenvalues of self-adjoint operators are real.