Last name ______________________

First name ______________________

LARSON—MATH 610–CLASSROOM WORKSHEET 24
Orthogonal Complements.

(Chp. 5). *inner product*, *inner product space*, $\langle\ \rangle$, *orthogonal* vectors, $\perp$, $||\cdot||$.
(Chp. 6). *orthogonal basis.*
(Chp. 7). *unitary matrix*, *unitarily similar.*
(Chp. 8). $U^{\perp}$, $V = U \oplus U^{\perp}$, *orthonormal projection*, $P_{\mathcal{U}}\hat{v}$.
(Chp. 9). eigenvalue, eigenvector, eigenpair, nullity, annihilating polynomial, spectrum.

Review:

1. (**Theorem 7.1.9**). Let $U, V \in \mathbb{M}_n$ and $W \in \mathbb{M}_m$.
   (a) If U and V are unitary then UV is unitary.
   (b) $V \oplus W$ is unitary if and only if V and W are unitary.
   (c) If U is unitary than $|\det(U)| = 1$.
2. When are matrices *similar*?
3. When are matrices *unitarily similar*?

Chp. 8 of Garcia & Horn, Matrix Mathematics

1. Let $\mathcal{V}$ be an inner product space and $\mathcal{U}$ be a subspace. What is $\mathcal{U}^{\perp}$?

2. What is an example?

(**Theorem 8.16**). Let $\mathcal{V}$ be an inner product space. If $\mathcal{U} = \mathcal{V}$ or $\mathcal{U}$ is a finite-dimensional subspace of $\mathcal{V}$ then $\mathcal{V} = \mathcal{U} \oplus \mathcal{U}^{\perp}$. In either case, for each $\hat{v} \in \mathcal{V}$ there is a unique $\hat{u} \in \mathcal{U}$ such that $\hat{v} - \hat{u} \in \mathcal{U}^{\perp}$.

3. What does this theorem say?

4. What is an example?

5. Why is this theorem true?

6. (**Theorem 8.3.10**). Let $\mathcal{U}$ be a finite-dimensional subspace of an inner product space $\mathcal{V}$.

   (a) $P_{\{\hat{0}\}} = 0$ and $P_{\mathcal{V}} = I$.

   (b) $ran(P_{\mathcal{U}}) = \mathcal{U}$.

   (c) $ker(P_{\mathcal{U}}) = \mathcal{U}^{\perp}$.

   (d) $\hat{v} - P_{\mathcal{U}}\hat{v} \in \mathcal{U}^{\perp}$ for all $\hat{v} \in \mathcal{V}$.

   (e) $||P_{\mathcal{U}}\hat{v}|| \leq ||\hat{v}||$ for all $\hat{v} \in \mathcal{V}$ with equality if and only if $\hat{v} \in \mathcal{U}$.

7. What does this theorem say?

8. Why is this theorem true?

Chp. 8 of Garcia & Horn, Matrix Mathematics

9. What is an *eigenvalue* and *eigenvector* of a matrix $A \in \mathbb{M}_n(\mathbb{F})$.

10. What is an example?