Last name ______________________

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LARSON—MATH 610—CLASSROOM WORKSHEET 22
Inner product Spaces.

Concepts & Notation

- (Chp. 5) *eigenvalue*, *eigenvector*, *invariant subspace*, *minimal polynomial*,
- (Chp. 8) *generalized eigenvector*, *Cayley-Hamilton Theorem*.
- (Chp. 6) *dot product*, *inner product*, *inner product space*, *norm*, *orthogonal representation*, *Cauchy-Schwartz*, *orthonormal list*, *Gram-Schmidt*, *orthogonal projection*.

Inner Product Spaces

1. (**Claim**) An orthonormal list of vectors in an inner product space is linearly independent.

2. What is the *Gram-Schmidt procedure*?

3. (**Existence of orthonormal basis**) Every finite-dimensional inner product space has an orthonormal basis.

4. (**Orthonormal list extends to orthonormal basis**) Suppose V is finite-dimensional. Then every orthonormal list of vectors in V can be extended to an orthonormal basis of V.

5. (**Upper-triangular matrix with respect to orthonormal basis**) Suppose $T \in \mathcal{L}(V)$ has an upper-triangular matrix with respect to some basis of V, then T has an upper-triangular matrix with respect to some orthonormal basis of V.

6. What is the *orthogonal complement* of a set U in an inner product space V?

7. (**Claim**:) If U is a subspace of an inner product space V then $V = U \oplus U^\perp$.

8. For a subspace U of an inner product space V, what is the *orthogonal projection* operator P_U?

9.

6.55 Properties of the orthogonal projection P_U

Suppose U is a finite-dimensional subspace of V and $v \in V$. Then

(a) $P_U \in \mathcal{L}(V)$;

(b) $P_U u = u$ for every $u \in U$;

(c) $P_U w = 0$ for every $w \in U^\perp$;

(d) range $P_U = U$;

(e) null $P_U = U^\perp$;

(f) $v - P_U v \in U^\perp$;

(g) $P_U{}^2 = P_U$;

(h) $\|P_U v\| \leq \|v\|$;

(i) for every orthonormal basis $e_1, \ldots, e_m$ of U,

$$P_U v = \langle v, e_1 \rangle e_1 + \cdots + \langle v, e_m \rangle e_m.$$