Last name ______________________

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LARSON—MATH 610—CLASSROOM WORKSHEET 21
Inner product Spaces.

Concepts & Notation

- (Chp. 5) *eigenvalue*, *eigenvector*, *invariant subspace*, *minimal polynomial*,
- (Chp. 8) *generalized eigenvector*, *Cayley-Hamilton Theorem.*
- (Chp. 6) *dot product*, *inner product*, *inner product space*, *norm*, *orthogonal representation*, *Cauchy-Schwartz*, *orthonormal list*, *Gram-Schmidt*, *orthogonal projection.*

Inner Product Spaces

1. What is an *orthonormal list* of vectors in an inner product space?
2. (**Claim**) An orthonormal list of vectors in an inner product space is linearly independent.
3. If $(e_1, \ldots, e_m)$ is an orthonormal list in an inner product space V (over $\mathbb{F}$) and $\alpha_1, \ldots, \alpha_m \in \mathbb{F}$ then $||\alpha_1 e_1 + \ldots \alpha_m e_m||^2 = |\alpha_1|^2 + \ldots + |\alpha_m|^2$.
4. What is an *orthonormal basis* in an inner product space?
5. If $e_1, \ldots, e_n$ is an orthonormal basis for an inner product space V, and $v \in V$, then
$$v = \langle v, e_1 \rangle e_1 + \ldots + \langle v, e_n \rangle e_n,$$
and
$$||v||^2 = |\langle v, e_1 \rangle|^2 + \ldots + |\langle v, e_n \rangle|^2.$$
6. What is the *Gram-Schmidt procedure*?
7. (**Existence of orthonormal basis**) Every finite-dimensional inner product space has an orthonormal basis.
8. (**Orthonormal list extends to orthonormal basis**) Suppose V is finite-dimensional. Then every orthonormal list of vectors in V can be extended to an orthonormal basis of V.

9. (**Upper-triangular matrix with respect to orthonormal basis**) Suppose $T \in \mathcal{L}(V)$ has an upper-triangular matrix with respect to some basis of V, then T has an upper-triangular matrix with respect to some orthonormal basis of V.

10. What is the *orthogonal complement* of a set U in an inner product space V?

11. (**Claim**:) If U is a subspace of an inner product space V then $V = U \oplus U^{\perp}$.

12. For a subspace U of an inner product space V, what is the *orthogonal projection* operator P_U?