Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 20
Inner product Spaces.

Concepts & Notation

- (Chp. 5) *eigenvalue*, *eigenvector*, *invariant subspace*, *minimal polynomial*,
- (Chp. 8) *generalized eigenvector*, *Cayley-Hamilton Theorem.*
- (Chp. 6) *dot product*, *inner product*, *inner product space*, *norm.*

Inner Product Spaces

1. What is the *orthogonal representation* of vectors u, v in an inner product space?
2. What is the *Pythagorean Theorem* for an inner product space?
3. What is the *Cauchy-Schwartz Inequality* in a inner product space?
4. What is an *orthonormal list* of vectors in an inner product space?

5. (**Claim**) An orthonormal list of vectors in an inner product space is linearly independent.

6. If $(e_1, \ldots, e_m)$ is an orthonormal list in an inner product space V (over $\mathbb{F}$) and $\alpha_1, \ldots, \alpha_m \in \mathbb{F}$ then $||\alpha_1 e_1 + \ldots \alpha_m e_m||^2 = |\alpha_1|^2 + \ldots + |\alpha_m|^2$.

7. What is an *orthonormal basis* in an inner product space?

8. If $e_1, \ldots, e_n$ is an orthonormal basis for an inner product space V, and $v \in V$, then

$$v = \langle v, e_1 \rangle e_1 + \ldots + \langle v, e_n \rangle e_n,$$

and

$$||v||^2 = |\langle v, e_1 \rangle|^2 + \ldots + |\langle v, e_n \rangle|^2.$$

9. What is the *Gram-Schmidt procedure*?

10. (**Existence of orthonormal basis**) Every finite-dimensional inner product space has an orthonormal basis.

11. (**Orthonormal list extends to orthonormal basis**) Suppose V is finite-dimensional. Then every orthonormal list of vectors in V can be extended to an orthonormal basis of V.

12. (**Upper-triangular matrix with respect to orthonormal basis**) Suppose $T \in \mathcal{L}(V)$ has an upper-triangular matrix with respect to some basis of V, then T has an upper-triangular matrix with respect to some orthonormal basis of V.