Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 19
Inner product Spaces.

Concepts & Notation

- (Chp. 5) *eigenvalue*, *eigenvector*, *invariant subspace*, *minimal polynomial*,
- (Chp. 8) *generalized eigenvector*, *Cayley-Hamilton Theorem.*
- (Chp. 6) *dot product*, *inner product*, *inner product space*, *norm.*

Inner Product Spaces

1. What is an *inner product* in a vector space?
2. What is an *inner product space*?
3. Let V be an inner product space, and $v \in V$. Check that $\langle 0, v\rangle = 0$ and $\langle v, 0\rangle = 0$.
4. Let V be an inner product space, and $u, v \in V$. Check that $\langle u, \alpha v\rangle = \overline{\alpha}\langle u, v\rangle$.
5. Let V be an inner product space, and $u, v, w \in V$. Check that $\langle u, v + w\rangle = \langle u, v\rangle + \langle u, w\rangle$.
6. Let V be an inner product space, what is the *norm* of $v \in V$?
7. Let V be an inner product space. What does it mean for vectors $u, v \in V$ to be *orthogonal*?
8. What is the *orthogonal representation* of vectors u, v in an inner product space?
9. What is the *Pythagorean Theorem* for an inner product space?
10. What is the *Cauchy-Schwartz Inequality* in a inner product space?

11. What is an *orthonormal list* of vectors in an inner product space?

12. (**Claim**) An orthonormal list of vectors in an inner product space is linearly independent.

13. If $(e_1, \ldots, e_m)$ is an orthonormal list in an inner product space V (over $\mathbb{F}$) and $\alpha_1, \ldots, \alpha_m \in \mathbb{F}$ then $||\alpha_1 e_1 + \ldots \alpha_m e_m||^2 = |\alpha_1|^2 + \ldots + |\alpha_m|^2$.

14. What is an *orthonormal basis* in an inner product space?

15. If $e_1, \ldots, e_n$ is an orthonormal basis for an inner product space V, and $v \in V$, then

$$v = \langle v, e_1 \rangle e_1 + \ldots + \langle v, e_n \rangle e_n,$$

and

$$||v||^2 = |\langle v, e_1 \rangle|^2 + \ldots + |\langle v, e_n \rangle|^2.$$