Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 18
Inner product Spaces.

Concepts & Notation

- (Chp. 5) *eigenvalue*, *eigenvector*, *invariant subspace*, *minimal polynomial*,
- (Chp. 8) *generalized eigenvector*, *Cayley-Hamilton Theorem*.
- (Chp. 6) *dot product*, *inner product*, *inner product space*.

Inner Product Spaces

1. What is the *dot product* of vectors in $\mathbb{R}^n$?

2. What is an *inner product* in a vector space?

3. Check that the dot product in $\mathbb{R}^2$ is an inner product.

4. What is an *inner product space*?

5. Let V be an inner product space, and $v \in V$. Check that $\langle 0, v\rangle = 0$ and $\langle v, 0\rangle = 0$.

6. Let V be an inner product space, and $u, v \in V$. Check that $\langle u, \alpha v \rangle = \overline{\alpha} \langle u, v \rangle$.

7. Let V be an inner product space, and $u, v, w \in V$. Check that $\langle u, v + w \rangle = \langle u, v \rangle + \langle u, w \rangle$.

8. Let V be an inner product space, what is the *norm* of $v \in V$?

9. Let V be an inner product space. What does it mean for vectors $u, v \in V$ to be *orthogonal*?

10. What is the *orthogonal representation* of vectors u, v in an inner product space?

11. What is the *Pythagorean Theorem* for an inner product space?

12. What is the *Cauchy-Schwartz Inequality* in a inner product space?