Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 16
Generalized Eigenvectors.

Concepts & Notation

- (Chp. 5) *eigenvalue*, *eigenvector*, *invariant subspace*, *minimal polynomial*,
- (Chp. 8) *generalized eigenvector*.

Complex Vector Spaces

1. Suppose $T : \mathbb{C}^3 \to \mathbb{C}^3$, with $T(x_1, x_2, x_3) = (4x_1, 0, 5x_2)$. Find all eigenvalues and associated eigenvectors.

2. Find the eigenspaces corresponding to the eigenvalues of T and check that they do not sum to $\mathbb{C}^3$.

3. What is a *generalized eigenvector*?

4. Find the generalized eigenvectors for $T : \mathbb{C}^3 \to \mathbb{C}^3$, with $T(x_1, x_2, x_3) = (4x_1, 0, 5x_2)$.

5. For each eigenvalue λ of T find the corresponding set G_λ of generalized eigenvectors of T.

6. Show that there is a basis of $\mathbb{C}^3$ consisting of generalized eigenvectors of T.

7. Show that $\mathbb{C}^3$ is a direct sum of the generalized eigenspaces corresponding to the eigenvalues of T.

8. Check that the generalized eigenspaces G_i are invariant under T.

9. For eigenvalues $\lambda_1, \ldots, \lambda_k$ of T, and generalized eigenspaces $G_1, \ldots, G_k$, let $d_i = dim\ G_i$ (d_i is the *multiplicity* of λ_i). Check that $d_1 = \ldots + d_k = dim(\mathbb{C}^3)$.

10. Find the characteristic polynomial $q(x) = (x - \lambda_1)^{d_1} \ldots (x - \lambda_k)^{d_k}$.

11. Check that $q(T) = 0$.

12. Explain why the minimal polynomial $p(x)$ of T must divide the characteristic polynomial $q(x)$ of T. Use this fact and other facts we proved to make conclusions about the minimal polynomial of T.