Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 12
Eigenvalues and Eigenvectors.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum*.
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*, *basis*.
- (Chp. 3) *linear map*, *null space*, *range*, *injective*, *surjective*, *invertible*, *isomorphism*, , *isomorphism*.
- (Chp. 4) *polynomial*, *root*.
- (Chp. 5) *eigenvalue*, *eigenvector*, *invariant subspace*.

Eigenvalues and Eigenvectors

1. What is an *eigenvalue* of $T \in \mathcal{L}(V)$?

2. (**Claim**:) λ is an eigenvalue of T if and only if $T - \lambda I$ is not injective.

3. What is an *eigenvector* of $T \in \mathcal{L}(V)$?

4. (**Claim**:) The set of eigenvectors corresponding to an eigenvalue of a linear operator $T \in \mathcal{L}(V)$ is $null(T - \lambda I)$ (and thus is a subspace of V).

5. Find the eigenvalues and eigenvectors for $T \in \mathcal{L}(\mathbb{R}^2)$ defined by $T(w, z) = (-z, w)$.

6. **Linearly independent eigenvectors**: Suppose $T \in \mathcal{L}(V)$. Every list of eigenvectors of T corresponding to distinct eigenvalues is linearly independent.

7. (**Claim**:) A finite-dimensional vector space V has at most $dim\ V$ eigenvalues.

8. If $T \in \mathcal{L}(V)$ and $p \in \mathcal{P}(\mathbb{F})$, what is $p(T)$?

9. (**Claim**:) Every operator on a finite-dimensional, nonzero, complex vector space has an eigenvalue.

10. What is an *invariant subspace* of $T \in \mathcal{L}(V)$?

11. For $T \in \mathcal{L}(V)$ the *eigenspace* $U = \{v : T(v) = \lambda v\}$ corresponding to an eigenvalue λ of T is an invariant subspace of T.

12. (**Claim**:) Suppose $T \in \mathcal{L}(V)$ and $(v_1, \ldots, v_n)$ is a basis of V. Then the following are equivalent:

    (a) the matrix of T with respect to $(v_1, \ldots, v_n)$ is upper-triangular;

    (b) $T(v_k) \in span(v_1, \ldots, v_k)$ for each $k = 1, \ldots, n$;

    (c) $span(v_1, \ldots, v_k)$ is invariant under T for each $k = 1, \ldots, n$.