Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 10
Polynomials and Eigenvalues.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum*.
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*, *basis*.
- (Chp. 3) *linear map*, *null space*, *range*, *injective*, *surjective*, *invertible*, *isomorphism*, , *isomorphism*.
- (Chp. 4) *polynomial*, *root*.
- (Chp. 5) *eigenvalue*.

Polynomials!

1. What is a *root* λ of a polynomial $p \in \mathcal{P}(\mathbb{F})$?
2. What is the Fundamental Theorem of Algebra?
3. What is the *degree* of a polynomial $p \in \mathcal{P}(\mathbb{F})$?
4. **Claim**: For polynomial $p \in \mathcal{P}(\mathbb{F})$ with degree $m \geq 1$, λ is a root of p if and only if there is a $q \in \mathcal{P}(\mathbb{F})$ with degree $m-1$ such that $p(z) = (z-\lambda)q(z)$ for every $z \in \mathbb{F}$.
5. **Division Algorithm**: If $p, q \in \mathcal{P}(\mathbb{F})$, $p \neq 0$, there are polynomials $s, r \in \mathcal{P}(\mathbb{F})$ such that $q = sp + r$ and $deg\ r < deg\ p$.
6. **Notation:** If $z \in \mathbb{C}$, what is: $Re(z)$, $Im(z)$? What is $\bar{z}$?
7. **Claim**: If $\lambda \in \mathbb{C}$ is a root of $p \in \mathbb{R}$ then so is $\bar{\lambda}$.

8. **Claim**: If $p \in \mathcal{P}(\mathbb{R})$ then p has a unique factorization:

$$p(x) = c(x - \lambda_1) \dots (x - \lambda_m)(x^2 + \alpha_1 x + \beta_1) \dots (x^2 + \alpha_M x + \beta_M).$$

with $\alpha_j^2 < 4\beta_j$.

Eigenvalues and Eigenvectors

9. What is an *eigenvalue* of $T \in \mathcal{L}(V)$?

10. What is an example of a linear operator with a real eigenvalue?

11. (**Claim**:) λ is an eigenvalue of T if and only if $T - \lambda I$ is not injective.

12. What is an *eigenvector* of $T \in \mathcal{L}(V)$?

13. (**Claim**:) The set of eigenvectors corresponding to an eigenvalue of a linear operator $T \in \mathcal{L}(V)$ is $null(T - \lambda I)$ (and thus is a subspace of V).

14. Find the eigenvalues and eigenvectors for $T \in \mathcal{L}(\mathbb{R}^2)$ defined by $T(w, z) = (-z, w)$.