Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 09
Isomorphisms.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum*.
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*, *basis*.
- (Chp. 3) *linear map*, *null space*, *range*, *injective*, *surjective*.

1. What is an *invertible* linear map?

2. **Notation**: If $T \in \mathcal{L}(V, W)$, and T is invertible, what is T^{-1}?

3. **Claim:** $T \in \mathcal{L}(V, W)$ is invertible if and only if T is injective and surjective.

4. What is a vector space *isomorphism*?

5. What does it mean for vector spaces V and W to be *isomorphic*?

Polynomials!

6. What is a *root* λ of a polynomial $p \in \mathcal{P}(\mathbb{F})$?

7. What is the Fundamental Theorem of Algebra?

8. What is the *degree* of a polynomial $p \in \mathcal{P}(\mathbb{F})$?

9. **Claim**: For polynomial $p \in \mathcal{P}(\mathbb{F})$ with degree $m \geq 1$, λ is a root of p if and only if there is a $q \in \mathcal{P}(\mathbb{F})$ with degree $m-1$ such that $p(z) = (z-\lambda)q(z)$ for every $z \in \mathbb{F}$.

10. **Division Algorithm**: If $p, q \in \mathcal{P}(\mathbb{F})$, $p \neq 0$, there are polynomials $s, r \in \mathcal{P}(\mathbb{F})$ such that $q = sp + r$ and $deg\ r < deg\ p$.

11. **Notation:** If $z \in \mathbb{C}$, what is: $Re(z)$, $Im(z)$? What is $\bar{z}$?

12. **Claim**: If $\lambda \in \mathbb{C}$ is a root of $p \in \mathbb{R}$ then so is $\bar{\lambda}$.

13. **Claim**: If $p \in \mathcal{P}(\mathbb{R})$ then p has a unique factorization:

$$p(x) = c(x-\lambda_1)\dots(x-\lambda_m)(x^2+\alpha_1 x+\beta_1)\dots(x^2+\alpha_M x+\beta_M).$$

with $\alpha_j^2 < 4\beta_j$.