Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 08
Linear Maps.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum.*
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*, *basis.*
- (Chp. 3) *linear map*, *null space*, *range*, *injective*, *surjective.*

The Matrix of a Linear Map

1. For $T \in \mathcal{L}(V, W)$, what is $\mathcal{M}(T, (v_1, \dots, v_n), (w_1, \dots, w_m))$?
2. For $T \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$ with $T(x, y) = (x + 3y, 2x + 5y, 7x + 9y)$, find $\mathcal{M}(T)$.
3. For $T \in \mathcal{L}(V, W)$, with basis $v_1, \dots, v_n$ for V and $w_1, \dots, w_m$ for W, $c \in \mathbb{F}$, check that:
$$\mathcal{M}(T + S) = \mathcal{M}(T) + \mathcal{M}(S),$$
$$\mathcal{M}(cT) = c\mathcal{M}(T).$$
4. What are the standard definitions of matrix addition, scalar multiplication, and matrix multiplication?
5. What is $\text{Mat}(m, n, \mathbb{F})$?
6. For vector spaces V, W, U, and linear maps $S : U \to V$ and $T : V \to W$, what is TS?

7. **Claim:** For vector spaces V, with basis $(v_1, \ldots, v_n)$, W, with basis $(w_1, \ldots, w_m)$ and U, with basis $(u_1, \ldots, u_p)$, and linear maps $S : U \to V$ and $T : V \to W$, we have:

$$\mathcal{M}(TS) = \mathcal{M}(T)\mathcal{M}(S).$$

8. If $x = (x_1, \ldots, x_n) \in \mathbb{F}^n$, what is $\mathcal{M}(x)$?

9. If $v \in V$, with basis $v_1, \ldots, v_n$, what is $\mathcal{M}(v)$?

10. **Claim**: If $T \in \mathcal{L}(V, W)$, $(v_1, \ldots, v_n)$ is a basis for V, $(w_1, \ldots, w_m)$ is a basis for W, then:

$$\mathcal{M}(Tv) = \mathcal{M}(T)\mathcal{M}(v),$$

for every $v \in V$.

11. What is an *invertible* linear map?

12. **Notation**: If $T \in \mathcal{L}(V, W)$, what is T^{-1}?

13. **Claim:** $T \in \mathcal{L}(V, W)$ is invertible if and only if T is injective and surjective.

14. What is a vector space *isomorphism*?

15. What does it mean for vector spaces V and W to be *isomorphic*?