Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 07
Linear Maps.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum.*
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*, *basis.*
- (Chp. 3) *linear map*, *null space*, *range*, *injective*, *surjective.*

Review

1. Let $T \in \mathcal{L}(V, W)$. **Claim:** *null* T is a subspace of V.
2. Let $T \in \mathcal{L}(V, W)$. **Claim:** *range* T is a subspace of W.
3. (**Rank-nullity Theorem**) If V is finite-dimensional and $T \in \mathcal{L}(V, W)$, then *range* T is finite-dimensional and

$$dim\ V = dim\ null\ T + dim\ range\ T.$$

The Matrix of a Linear Map

4. For $T \in \mathcal{L}(V, W)$, what is $\mathcal{M}(T, (v_1, \ldots, v_n), (w_1, \ldots, w_m))$?

5. For $T \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$ with $T(x, y) = (x + 3y, 2x + 5y, 7x + 9y)$, find $\mathcal{M}(T)$.

6. For $T \in \mathcal{L}(V, W)$, with basis $v_1, \ldots, v_n$ for V and $w_1, \ldots, w_m$ for W, $c \in \mathbb{F}$, check that:

$$\mathcal{M}(T + S) = \mathcal{M}(T) + \mathcal{M}(S),$$

$$\mathcal{M}(cT) = c\mathcal{M}(T).$$

7. What is $\mathrm{Mat}(m, n, \mathbb{F})$?

8. What are the standard definitions of matrix addition, scalar multiplication, and matrix multiplication?

9. For vector spaces V, W, U, and linear maps $S : U \to V$ and $T : V \to W$, what is TS?

10. **Claim:** For vector spaces V, with basis $(v_1, \ldots, v_n)$, W, with basis $(w_1, \ldots, w_m)$ and U, with basis $(u_1, \ldots, u_p)$, and linear maps $S : U \to V$ and $T : V \to W$, we have:

$$\mathcal{M}(TS) = \mathcal{M}(T)\mathcal{M}(S).$$

11. If $x = (x_1, \ldots, x_n) \in \mathbb{F}^n$, what is $\mathcal{M}(x)$?

12. **Claim**: If $T \in \mathcal{L}(V, W)$, $(v_1, \ldots, v_n)$ is a basis for V, $(w_1, \ldots, w_m)$ is a basis for W, then:

$$\mathcal{M}(Tv) = \mathcal{M}(T)\mathcal{M}(v),$$

for every $v \in V$.