Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 06
Linear Maps.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum.*
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*, *basis.*
- (Chp. 3) *linear map*, *null space*, *range*, *injective*, *surjective.*

Review

1. (**Linear map lemma**.) If $v_1, \ldots, v_n$ is a basis for vector space V and $w_1, \ldots, w_n$ is a basis for vector space W then there is a unique linear map $T : V \to W$ with $Tv_i = w_i$.

2. Let $T \in \mathcal{L}(V, W)$. What is the *null space* of T? (**Notation:** *null* T).

3. Let $T \in \mathcal{L}(V, W)$. **Claim:** *null* T is a subspace of V.

4. Let $T \in \mathcal{L}(V, W)$. What does it mean for T to be *injective.*

New

5. Let $T \in \mathcal{L}(V, W)$. T is injective if and only if *null* $T = \{0\}$.

6. Let $T \in \mathcal{L}(V, W)$. What is the *range* of T? (**Notation:** *range* T).

7. What do we say if *range* $T = W$?

8. Let $T \in \mathcal{L}(V, W)$. **Claim:** *range* T is a subspace of W.

9. (**Rank-nullity Theorem**) If V is finite-dimensional and $T \in \mathcal{L}(V, W)$, then *range* T is finite-dimensional and

$$dim\ V = dim\ null\ T + dim\ range\ T.$$

The Matrix of a Linear Map

10. For $T \in \mathcal{L}(V, W)$, what is $\mathcal{M}(T, (v_1, \ldots, v_n), (w_1, \ldots, w_m))$?

11. For $T \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$ with $T(x, y) = (x + 3y, 2x + 5y, 7x + 9y)$, find $\mathcal{M}(T)$.

12. For $T \in \mathcal{L}(V, W)$, with basis $v_1, \ldots, v_n$ for V and $w_1, \ldots, w_m$ for W, $c \in \mathbb{F}$, check that:

$$\mathcal{M}(T + S) = \mathcal{M}(T) + \mathcal{M}(S),$$
$$\mathcal{M}(cT) = c\mathcal{M}(T).$$