Last name ______________________

First name ______________________

LARSON—MATH 610—CLASSROOM WORKSHEET 04
Review.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum.*
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*, *basis.*

Review

1. (**Linear Dependence Lemma**) If $v_1, \ldots, v_m$ in V are linearly dependent, then:
   (a) $\exists j \in \{1, \ldots, m\}$ $v_j \in span(v_1, \ldots, v_{j-1})$.
   (b) $span(v_1, \ldots, v_m) = span(v_1, \ldots, \hat{v}_j, \ldots, v_m)$.

2. **Theorem:** In a finite-dimensional vector space, the length of every linearly independent list of vectors is no more than the length of every spanning list of vectors.

3. (**Basis Criterion**). A list $v_1 \ldots ; v_n$ of vectors in a vector space V is a basis of V if and only if every $v \in V$ can be written uniquely in the form $v = a_1 v_1 + \ldots + a_n v_n$ $(a_i \in \mathbb{F})$.

New Material

4. (**Spanning list contains a basis**). Every spanning list in a vector space can be reduced to a basis of the vector space.

5. (**Basis of finite-dimensional vector space**). Every finite-dimensional vector space has a basis.

6. (**Linearly independent list extends to a basis**). Every linearly independent list of vectors in a finite-dimensional vector space can be extended to a basis of the vector space.

7. (**Basis length is independent of basis**). Any two bases of a finite-dimensional vector space have the same length.

8. What is the *dimension* of a finite-dimensional vector space?

9. **Claim:** If V is a finite-dimensional vector space with $dim(V) = n$ then any linearly independent list of n vectors is a basis of V.