Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 02
Review.

Concepts & Notation

- (Chp. 1) *field* $\mathbb{F}$, *list*, *vector space*, $\mathbb{F}^n$, $\mathbb{F}^S$, $\mathbb{F}^\infty$, *subspace*, sums of subspaces, *direct sum*.
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*.

1. When is $U_1 + \ldots + U_m$ a *direct sum*? What is the notation?

2. What is a *linear combination* of vectors $v_1, \ldots, v_m$ (over a field $\mathbb{F}$)?

3. What is the *span* of vectors $v_1, \ldots, v_m$ (over a field $\mathbb{F}$)?

4. (**Claim**). The span of vectors $v_1, \ldots, v_m$ in V is a subspace of V?

5. When is a vector space V *finite-dimensional*?

6. What is a *polynomial* function $p : \mathbb{F} \to \mathbb{F}$?

7. What is $\mathcal{P}(\mathbb{F})$?

8. What is a *linearly independent* list of vectors?

9. What is a *linearly dependent* list of vectors?

10. (**Linear Dependence Lemma**) If $v_1, \ldots, v_m$ in V are linearly dependent, then:
    (a) $\exists j \in \{1, \ldots, m\}$ $v_j \in span(v_1, \ldots, v_{j-1})$.
    (b) $span(v_1, \ldots, v_m) = span(v_1, \ldots, \hat{v_j}, \ldots, v_m)$.

11. **Claim:** In a finite-dimensional vector space, the length of every linearly independent list of vectors is no more than the length of every spanning list of vectors.