

Last name _____

First name _____

LARSON—MATH 610—CLASSROOM WORKSHEET 01
Review.

Concepts & Notation

- (Chp. 1) *field* \mathbb{F} , *list*, *vector space*, \mathbb{F}^n , \mathbb{F}^S , \mathbb{F}^∞ , *subspace*, sums of subspaces, *direct sum*.
- (Chp. 2) *linear combination*, *span*, *finite-dimensional* vector space, *linear independence*.

1. What is a field \mathbb{F} ?
2. What is a *vector space*?
3. What is \mathbb{F}^S , for a set S ?
4. What is \mathbb{F}^∞ ?
5. What is a *subspace* of a vector space V ?
6. What is $U_1 + \dots + U_m$ for subspaces U_1, \dots, U_m of V ?
7. When is $U_1 + \dots + U_m$ a *direct sum*? What is the notation?

8. What is a *linear combination* of vectors v_1, \dots, v_m (over a field \mathbb{F})?

9. What is the *span* of vectors v_1, \dots, v_m (over a field \mathbb{F})?

10. (**Claim**). The span of vectors v_1, \dots, v_m in V is a subspace of V ?

11. When is a vector space V *finite-dimensional*?

12. What is a *polynomial* function $p : \mathbb{F} \rightarrow \mathbb{F}$?

13. What is $\mathcal{P}(\mathbb{F})$?

14. What is a *linearly independent* list of vectors?

15. What is a *linearly dependent* list of vectors?

16. (**Linear Dependence Lemma**) If v_1, \dots, v_m in V are linearly dependent, then:
 - (a) $\exists j \in \{1, \dots, m\} \ v_j \in \text{span}(v_1, \dots, v_{j-1})$.
 - (b) $\text{span}(v_1, \dots, v_m) = \text{span}(v_1, \dots, \hat{v}_j, \dots, v_m)$.