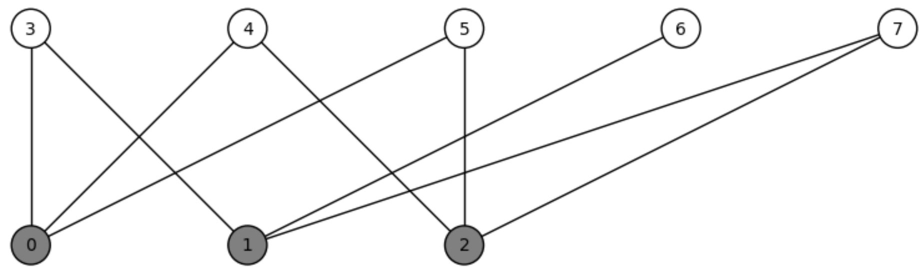
Last name ______________________

First name ______________________

LARSON—MATH 556–HOMEWORK WORKSHEET 12
Network Flow & Bipartite Matching



Let the bipartite graph above be $G = (A, B)$ where A is the set of white points.

1. Draw a network (D,s,t,c) by adding a point s and directing an edge from s to each point in A. Add a direction of the edges in G in the direction from A to B. Add a point t and direct an edge from each point in B to t. Make the capacities c of all edges incident to s or t equal to 1, and the capacities of all the edges from A to B equal ∞.
2. Find a maximum *integer* flow f in (D,s,t,c). That is, for each directed edge (x, y), $f(x, y)$ is an integer (and so, in this case, either $f(x, y) = 0$ or $f(x, y) = 1$).
3. Explain why f satisfies the conditions of a flow.
4. Find $val(f)$.
5. Argue that the flow you found is indeed maximum by finding A_f and the capacity of the cut $\nabla^+(A_f)$.
6. Find $\nu(G)$.
7. Find the lines from G which have positive flow.
8. Why *must* the lines in G with positive flow constitute a matching?
9. Why *must* the lines in G with positive flow constitute a *maximum* matching?
10. *How* can the cut $\nabla^+(A_f)$ can be used to define a minimum vertex cover in G?
11. Prove: for any bipartite graph $G = (A, B)$ with corresponding network defined as in our example, and maximum integer flow f, that $val(f) = \nu(G)$.