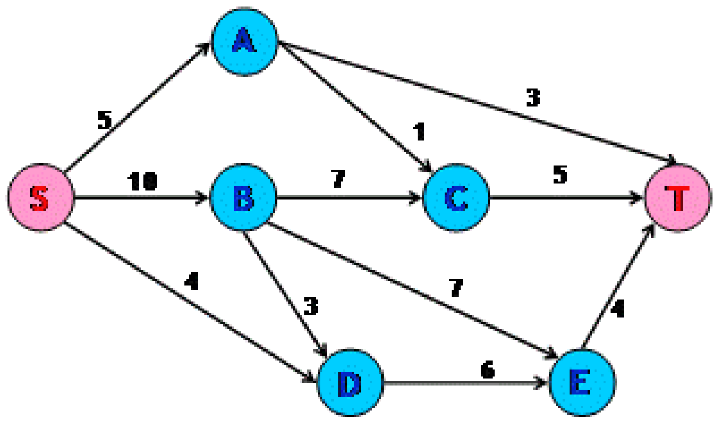
Last name ______________________

First name ______________________

LARSON—MATH 556–HOMEWORK WORKSHEET 11
Network Flow & Integrality of Flow



S is the source and T is the sink of this network. The edges are labeled with their capacities. Let f_0 be the zero-flow (that is, flow is zero on all directed lines) and $val(f_0) = 0$.

1. Find, and indicate a flow-augmenting path P.

2. Find ϵ_1, ϵ_2, and ϵ (as in the proof that a flow is maximum iff there are no flow-augmenting paths). Note that, since the capacities are integers that ϵ is necessarily an integer. ϵ is the most this flow can be increased along path P.

3. Define a new flow f_1 by increasing the flow along P by ϵ. Remember to check that your flow satisfies the 2 flow constraints. Draw a new diagram indicating the flow values. Find $val(f_1)$.

4. If there is another flow-augmenting path P, find it and repeat: find ϵ, define an improved flow f_2, find $val(f_2)$, and draw a new diagram. Keep repeating this process until there are no longer any flow-augmenting paths.

5. Let your final flow be f. Find $val(f)$. Note that it must be an integer.

6. Find A_f (also as in the proof that a flow is maximum iff there are no flow-augmenting paths). A_f is a set of points.

7. Find the cut $C = \nabla^+(A_f)$ (this is a set of directed lines, If you have done everything correctly the flow on these lines will equal their capacities, and the flow on the lines from $V - A_f$ to A_f will be zero).

8. Find cap(C). If you have done everything correctly $val(f) = cap(C)$!